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Multivariate GARCH Models for the Greater China Stock Markets

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Abstract

This paper reviews the commonly used multivariate GARCH models and uses the daily data of the four Greater China region stock markets, namely Hongkong, Shanghai,Shenzhen, and Singapore, and data of Japan as one exogenous variable to investigate the volatility and shocks spillover behavior and to establish the market linkage among the four markets. We find that the volatility spillover between Shanghai and Shenzhen is obvious and correlation contagion is detected. Conditional variance and conditional correlations are time varying and dynamic which conforms to the arguments in most of the literature. Shanghai and Shenzhen present a very high correlation level during the sampling period,varying from 0.75 to 0.98, at some point even near linear correlation, which is not uncommon due to the close interlink between the two markets. Hongkong and Singapore presents a mildly high correlation, varying from 0.25 to 0.9, with an average of 0.62. However, the correlation is very volatile. Results present the convincing evidence that Chinese stock markets are more and more integrated to the global markets and the Greater China region markets are more integrated to each other. There are many obvious correlation breaks,when all the correlations suddenly drop to a drastically low level. The drop corresponds to the actual economic event as we discover.

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1 Introduction

Volatilities and correlations are the two most important elements in asset pricing, portfolio management and risk assessment. Since the seminal 1982 paper of Engel's autoregressive conditional heteroskedasticity (ARCH) model, lots of efforts have been spent on univariate volatility modeling. Most famous one among them is the Bollerslov's generalized ARCH (GARCH) model. As time goes by and computing power improves, researchers find it more and more important and necessary to generalize the univariate ARCH/GARCH models to their multivariate versions. This will continue to be the trend thereafter.

One of the central aspects in financial econometrics is the modeling, measuring and forecasting of second and possible higher moments, because the volatility for instance is not directly observable. One of the most important models for volatility is the class of multivariate generalized autoregressive conditional heteroscedasticity (MARCH) models. They allow us to specify a dynamic process for the whole time varying variance–covariance matrix of the time series thus jointly modeling the first and second moments. The main applications of MARCH models are in portfolio management, hedging, analysis of volatility spillovers across markets, conditional CAPM, option pricing and Value–at–Risk (VaR) of portfolios. Since correlations between asset returns and markets are important in many financial applications, multivariate volatility models have also been extended to describe the time–varying feature of the correlations in recent years.

The univariate GARCH framework was developed by Bollerslev (1986), based on the ARCH models by Engle (1982). Engle proposed a function for the conditional variance of the time series that depends on the realized error of the period before. Analogous to the expansion from the AR models to ARMA models, Bollerslev developed the GARCH model by taking the own history of the volatility into account. But in this framework the restrictions to univariate time series doesn't take the volatility spillover into account. The possibility of interaction between one or more time series is completely excluded. Therefore Bollerslev, Engle and Wooldridge (1988) proposed the basic framework for the multivariate GARCH model by including additional parameters in order to capture this effect. Many expansions to the basic MARCH models have therefore been developed. Based on the recent theoretical and empirical developments and discoveries in MARCH models, this paper focuses on the investigation of volatilities and correlations of the stock markets in the so–called Greater China region, that is, Shanghai, Shenzhen, Hongkong and Singapore.

The paper is organized as follows. Section 2 reviews the developments and applications of MARCH models in recent decades. Section 3 introduces the popular econometric specifications of MARCH models, which form the basis of our empirical study in Section 5. We first review the basic forms. Then we present the extended and/or generalized versions. We give a brief description of the data employed in this paper in Section 4. Section 5 presents the estimation results and empirical findings. Finally, we conclude the paper in section 6 as well as some discussion of the weakness and possible future research directions of MARCH models.

2 Literature Review

In this section we review the development of multivariate GARCH models and their wide applications. Understanding and predicting volatilities and correlations of asset returns has been the object of much attention, since volatilities and correlations are the two most important elements in financial activities such as asset pricing, asset allocation decisions, portfolio management and rish assessment.

In the last few decades, so many volatility models have been put forward. The most popular and successful models among them are the autoregressive conditional heteroskedasticity (ARCH) model by Engle (1982) and extended to generalized ARCH (GARCH) model by Bollerslev (1986). The ARCH/GARCH models have generated a great spectrum of models, which have been applied and tested in many areas. Their success stems from their ability to capture some stylized facts of the studied time series, especially for financial time series, such as time–varying volatility and volatility clustering. See Bollerslev, Engle, and Nelson (1994), Bera and Higgins (1993), and Kroner and Ng (1998) for a comprehensive survey of the univariate volatility models and their application. See also Bollerslev, Chou, and Kroner (1992) for a review of the ARCH modeling in the financial area. Other volatility models include the vast stochastic volatility (SV) models and etc., which we will not explore here, however. See Taylor (1994) for a review of the univariate SV models. See Harvey, Ruiz, and Shephard (1994) for a review of the multivariate SV models.

Although univariate ARCH/GARCH models have been proved to be very powerful in explaining the stylized facts of univariate time series, researchers find them unsatisfactorily incapable to examine the characteristics of multivariate time series simultaneously. Since in reality we are more concerned about the relationships between volatilities of several markets or assets and variance–covariance matrices of various portfolios, univariate ARCH/GARCH models seem to be not applicable and therefore their multivariate generalization stands out to be the better solution. There are generally two directions for modeling the multivariate time series, modeling the variance–covariance matrix directly and modeling the correlation between the time series indirectly. Bollerslev, Engle, and Wooldridge (1988) proposed the first multivariate GARCH model for the conditional variance–covariance matrix, namely the VEC model, which was a successful attempt towards the first direction. However, this model is a very general model and very difficult to implement in practice. The number of parameters in the model is $O(K^4)$ with respect to the dimension of the model and it is difficult to impose the positive definiteness of the variance–covariance matrix in the model. Thus, a portion of the subsequent literature is to try to simplify this model. It should be noted that the advantage of this model is that we can directly interpret the coefficients in the model. Bollerslev, Engle, and Wooldridge (1988) introduced a simplified version of the VEC model, the Diagonal–VEC model. This model reduced the number of parameters greatly and it is relatively easier to derive the conditions to guarantee the positive definiteness of variance–covariance matrix. However, since the variance or covariance in the model is only the function of its past observations, it can not capture the interactions between different variances and covariances.

Engle and Kroner (1995) proposed the BEKK [1] model which can be viewed as a restricted version of VEC model. BEKK model has a very good property, that is, conditional variance–covariance matrix is positive definite by construction. But the number of parameters in BEKK model still increase rapidly with the dimension of the model. Another problem is that it is hard to interpret the coefficients of the model. Further simplified models include the Diagonal–BEKK model and the Scalar–BEKK model. Diagonal–BEKK model faces the same problem of Diagonal–VEC model, although it reduces the number of parameters greatly. Scalar–BEKK model is too restrictive as it imposes the same dynamics to all the variances and covariances.

Engle, Ng, and Rothschild (1990) developed another way to reduce the number of

[1] BEKK is the acronym of Baba, Engle, Kraft, and Kroner who initially wrote the paper.

parameters involved in the model by introducing several factors. The main problem of multivariate GARCH models in most specifications is the very large number of parameters, which rapidly makes the estimation infeasible as the number of series increases. Those specifications which bypass this problem, on the other hand, pay the price in terms of a severe loss of generality. Neither multivariate SV models, although relatively more parsimonious, are able to handle more than a few number of series because of their complexity of estimation. The key for dimensionality reduction stands in the idea of the existence of a few latent variables, the so called factors, as driving forces for the whole dataset. Back to finance, models as CAPM explain theoretically why we may speak of factors in the market. In this spirit, Engle, Ng, and Rothschild (1990) assumed that the relationships between different asset returns are driven by some factors which are conditionally heteroskedastic and possess a GARCH–type structure. The approach has the advantage that it can solve the problem of dimensionality by modeling the factors which is much less than the number of assets in terms of number. In their paper, the authors considered two factors, one is the value–weighted index return and the other one is the T–bill returns of different maturities. Alexander (2001) proposed the Orthogonal–GARCH model. The author used principle component analysis and constructed the unobserved uncorrelated factors which are assumed to have different univariate GARCH structures. The number of parameters can be reduced to $O(K)$, where K is the number of factors. However, one big disadvantage of this approach is that it is difficult to interpret the parameters as the BEKK model.

Another direction for MGARCH models is to model the correlation indirectly between the series instead of modeling the variance–covariance matrix directly. Bollerslev (1990) first introduced a class of constant conditional correlation (CCC) model in which conditional correlation matrix is assumed to be constant and thus the conditional covariances are proportional to the product of the corresponding conditional standard deviations. The specification of CCC model is innovative, because it has desirably fewer parameters, it saves a lot of computational cost as only one correlation matrix is needed to be inverted in each iteration using maximum likelihood method, and it automatically guarantees the positive definiteness of the variance–covariance matrix. But the assumption that conditional correlation matrix is time–invariant is unrealistic in many empirical applications.

Engle (2002) and Tse and Tsui (2002) generalized the CCC model to make the conditional correlation matrix time–varying. An additional difficulty for time–varying correlation models is that the time–varying conditional correlation matrix has to be positive definite for every time t. Engle (2002)'s dynamic conditional correlation (DCC) model specified a GARCH–type dynamic matrix process and then transformed the variance–covariance matrix to the correlation matrix. Alternatively, time–varying correlation (TVC) model of Tse and Tsui (2002) formulated the conditional correlation as a weighted sum of past correlations, where the conditional correlation matrix was assumed to resemble an ARMA structure. However, both models of Engle (2002) and Tse and Tsui (2002) lose computational efficiency, as T correlation matrices are needed to be inverted in each iteration using maximum likelihood method, where T is the number of observations. Another drawback of the DCC–type models is that it restricts all the correlation processes to obey the same dynamic structure. Interestingly, these models can be estimated consistently using two–step estimation.

Several variants of the DCC model are proposed in the literature. Billio, Caporin, and Gobbo (2003) argued that constraining the dynamics of the conditional correlation matrix to be the same for all the corrleations is not appealing. To overcome this issue, they proposed a block–diagonal structure, where the dynamics is constrained to be the same only within each block. However, the number of blocks has to be defined *a priori*, which may be tricky in some applications. Pelletier (2003) proposed a regime–switching DCC model, where the conditional correlations follow a switching regime and the correlation matrix is constant in each regime but may vary across regimes. This model is highly computational. Cappiello, Engle, and Sheppard (2006) advocated the asymmetric generalized dynamic conditional correlation (AG–DCC) model. The AG–DCC process allows for series–specific news impact and smoothing parameters and permits conditional asymmetries in correlation dynamics. The AG–DCC specification is well suited to examine correlation dynamics among different asset classes and investigate the presence of asymmetric responses in conditional variances and correlations to negative returns. Considering incorporating the prevalent asymmetric effects and possible block structures to avoid same dynamics for all assets in financial time series, Vargas (2006) proposed the asymmetric block dynamic conditional correlation (ABDCC) model. McAleer, Chan, Hoti, and Lieberman (2008) gave a generalized autoregressive conditional correlation model. Engle and Kelly (2008) developed the equicorrelation model, which is a highly simplified version of the DCC model. However, the equicorrelation assumption seems to be very restrictive and inadequate.

Now we turn to a summary for empirical applications of MARCH models. Many studies provide evidence that correlation is evolving through time. Longin and Solnik (1995) showed that correlation in international equity returns across 1960–1990 is highly volatile. Engle (2002) verified the important evidence of time–varying correlation of many classes of assets. Tse and Tsui (2002) applied time–varying correlation model to exchange rate date, national stock market data and the sectoral price data and provided the time–varying correlation evidence for the three real datasets. Solnik, Boucrelle, and Le Fur (1996) found that correlation is increasing in periods of high market volatility for the industrialized countries when risk diversification is needed most. Campbell, Koedijk and Kofman (2002) showed that market correlations increase in the bear market. Volatility changes not only due to the dynamic evolution of own market volatility but also changes of interdependence across markets. Hamao, Masulis, and Ng (1990) examined the combination of correlations in price changes and volatility across international stock markets. Engle and Susmel (1993) found that there is common volatility in international equity markets. Bollerslev and Engle (1993) checked the common persistence effect in the conditional variances, that is, the volatility. Bae and Karolyi (1994) found that the spillover of stock volatility between Japan and the United States is closely related to goods news or bad news. Karolyi (1995) used a bivariate GARCH model to investigate the transmission of stock returns and volatility between the United States and Canada, finding that volatility is transferred from U.S. to Canada most of the time. See King, Sentana and Wadhwani (1994), Lin, Engle, and Ito (1994) and Ng (2000) for more evidences of volatility transmission and linkage. Lanza, Manera, and McAleer (2006) and Manera, McAleer, and Grasso (2006) examined correlation and volatility in the oil forward and future markets. Edwards and Susmel (2001) and Edwards and Susmel (2003) investigated the volatility dependence and contagion in equity and interest rate respectively in emerging markets. Balasubramanyan and Premaratne (2003) and Balasubramanyan (2004) provided the evidence of volatility comovement and spillover from Asian markets. Yang (2005) used a DCC anaylsis to examine the role of Japan on the Asian Four Tigers, finding that stock market correlations fluctuate widely over time and volatilities are contagious across markets. Kuper and Lestano (2007) analyzed the financial market interdependence of Thailand and Indonesia. See Andersen, Bollerslev, Christoffersen, and Diebold (2005) for a review of volatility and correlation modeling for financial markets.

3 Econometric Methodology

3.1 Basics

This section begins with a short introduction to explain what we are going to model. The basic idea in volatility modeling is to decompose a given multivariate time series into a predictable and an unpredictable part. Many multivariate volatility models are available in the literature. We will only describe in detail those models that will be employed in the empirical analysis in this paper. Indeed, multivariate volatility models are in essence vector volatility models. Consider a stochastic vector process $\{r_t\}$ with dimension $N \times 1$. We denote by $\mathcal{F}_{t-1}$ the σ-field, which is the information set generated by the observed series $\{r_t\}$ up to and including time $t-1$ and denote by θ a finite vector of parameters. By convention, we assume that $\{r_t\}$ is conditionally heteroskedastic in the following way:

$$r_t = \mu_t(\theta) + \varepsilon_t \tag{1}$$

where $\mu_t(\theta)$ is the predictable conditional mean vector with respect to the information set $\mathcal{F}_{t-1}$ and

$$\varepsilon_t = H_t^{1/2}(\theta)\zeta_t \tag{2}$$

is the unpredictable errors given the information set $\mathcal{F}_{t-1}$, where the $N \times N$ positive definite and symmetric matrix $H_t(\theta) = [h_{ijt}(\theta)]$ is the conditional variance–covariance matrix of r_t. [2] Furthermore, we assume that the $N \times 1$ vector ζ_t is an i.i.d. random vector such that $E(\zeta_t) = 0$ and $E(\zeta_t \zeta_t') = I_N$, where I_N is an identity matrix of order N. Put it another way, $r_t | \mathcal{F}_{t-1} \sim U(\mu_t(\theta), H_t(\theta))$, where $U(\mu_t(\theta), H_t(\theta))$ is an un–specified multivariate distribution with time dependent mean $\mu_t(\theta)$ and time dependent variance–covariance matrix $H_t(\theta)$. The above formulation defines the stan-

[2] In practice, Cholesky decomposition can be applied to obtain $H_t^{1/2} H_t^{1/2} = H_t$.

dard multivariate GARCH framework with no linear dependence structure in $\{r_t\}$. This formulation actually nests all the multivariate GARCH representations that will be introduced in the next section, and allows also the specification of a multivariate ARMA process for the mean, as well as the GARCH-in-mean effects for the variance. However, to maintain simplicity, we leave out θ in the notations of conditional mean and conditional variance covariance matrix hereafter. Furthermore, we assume $\mu_t = 0$ without loss of generality hereafter, since the behavior of the conditional mean is relatively simple for asset returns and μ_t is simply a constant in most cases. In most financial applications, r_t is often viewed as a vector of log–returns (in percentages) of N assets denoted by $r_t = 100 \cdot (\log(P_t) - \log(P_{t-1}))$, where P_t represents the value of the indexed asset.

What remains to be specified is the matrix process H_t. How do we parameterize the matrix H_t proves to produce rather different results. Many times the different results present us different insights. Various parametric formulations have been proposed in the literature by now. Because of different directions and efforts in tackling the variance–covariance matrix H_t, two very general classes of models emerged in this sense, namely, modeling conditional covariance matrix H_t directly, e.g. VEC model or BEKK model and modeling conditional correlation matrix indirectly, e.g. constant conditional correlation (CCC) model or dynamic conditional correlation (DCC) model. These models and extensions will be examined separately in the following subsections. However, we will focus our attention on the parametric versions of MGARCH models while ignoring the vast literatures on non–parametric and semi–parametric versions and factor MARCH models. To have detailed ideas, we recommend the excellent surveys by Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2008).

3.2 Models of conditional covariance matrix

3.2.1 VEC–GARCH Model

The VEC model proposed by Bollerslev, Engle, and Wooldridge (1988) is a straightforward generalization of the univariate GARCH model to the multivariate case. Every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross–products of returns. [3] The specification can be written as follows: [4]

$$\mathrm{vech}(H_t) = \omega + \sum_{j=1}^{q} A_j \mathrm{vech}(\varepsilon_{t-j}\varepsilon_{t-j}^{'}) + \sum_{i=1}^{p} B_i \mathrm{vech}(H_{t-i}) \tag{3}$$

where ω is a $N(N+1)/2 \times 1$ vector, and A_j and B_i are $N(N+1)/2 \times N(N+1)/2$ parameter matrices. Due to the general form of the VEC representation, a wide range of multivariate dynamic structure is possible. One prominent advantage is that VEC model has explicit interpretation of the parameters. However, one of the central problems of the VEC model is its large number of parameters to be estimated. This is known to be the curse of dimensionality. The total number of parameters equals $(p+q)(N(N+1)/2)^2+N(N+1)/2$, which is undesirably large even when $p = q = 1$ and N is small. In order to estimate the higher dimensional VEC class of models, we must be blessed with great powerful computing facilities to maximize the likelihood and calculate the standard errors. We still have to estimate 21 parameters when $N = 2$, which simply is the bivariate case. In practice, the VEC models is only estimated when N is small and $p = q = 1$. Bollerslev, Engle, and Wooldridge (1988) simplified the VEC model to a diagonal version in order to make the estimation more applicable. In the diagonal version, parameter matrices A_j and B_i are diagonal. Nevertheless, we can immediately recognize that this simplification has one big disadvantage. It

[3] Note that we have assumed $\mu = 0$ and therefore $r_t = \varepsilon_t$.

[4] For a symmetric $N \times N$ matrix A, vech(A) is a $N(N+1)/2 \times 1$ vector, where vech($\cdot$) is an operator that stacks the columns of the lower triangular part of its argument square matrix.

ignores dynamics that allow volatility in one asset (market) spillovers on the volatility of other assets (markets),while in certain cases the volatility spillover may prove to be significant. Another problem with VEC model is that it's hard to insure the positive definiteness of the variance covariance matrix when high dimensional form is imposed. In our empirical analysis with four return series in Section 5, we will estimate both the general and diagonal forms.

3.2.2 BEKK–GARCH Model

BEKK model is in fact a restricted version of the VEC model. It's been designed to ensure the positive definiteness of the variance covariance matrix H_t. The basic BEKK presentation has been proposed by Engle and Kroner (1995) in the following way:

$$H_t = \Omega\Omega' + \sum_{j=1}^{q} A_j \varepsilon_{t-j} \varepsilon_{t-j}^{'} A_j^{'} + \sum_{i=1}^{p} B_i H_{t-i} B_i^{'} \tag{4}$$

where A_j, B_i and Ω are all $N \times N$ parameter matrices, and Ω is lower triangular matrix. The decomposition of the constant term into a product of two triangular matrices is to ensure the positive definiteness of variance covariance matrix H_t. Engle and Kroner (1995) show that BEKK model is covariance stationary if and only if the eigenvalues of $\sum_{j=1}^{q} A_j \otimes A_j + \sum_{i=1}^{p} B_i \otimes B_i$ are less than one in modulus, where $\otimes$ denotes the Kronecker product of two matrices. Whenever $K > 1$,an identification problem arises because there are several parameterizations that yield the same representation of the model. Engle and Kroner (1995) give conditions for eliminating redundant, observationally equivalent representations.

Still, how to interpret the parameters of equation 4 is a demanding job. Curse of dimensinality is also an issue for BEKK model. Let's restrict ourselves to the first order model

$$H_t = \Omega\Omega' + A\varepsilon_{t-1}\varepsilon_{t-1}^{'} A' + BH_{t-1}B' \tag{5}$$

Empirical applications often involve the highly simplified version of equation 5 when both A and B are assumed to be diagonal matrices. This is called diagonal BEKK, proposed by Bollerslev, Engle, and Wooldridge (1988). The main advantage is that the number of parameters decreases to $N(N+1)/2+2N$ while still maintaining the positive definiteness of H_t.

3.3 Models of conditional correlation matrix

3.3.1 CCC–GARCH Model

In the past years, a new class of multivariate GARCH models has been developed. They focus on the parametrization of the conditional correlation matrix. Such models have the flexibility of univariate GARCH models with respect to the conditional variances. They need simple conditions to ensure the positive definiteness of H_t and the estimation is much easier than the usual MARCH models. The constant conditional correlation (CCC) model of Bollerslev (1990) is a fruitful endeavor to explore the MGARCH model indirectly in the correlation direction instead of modeling the variance covariance matrix H_t directly. CCC model has several advantages mentioned above. Now we define the structure of the constant conditional correlation matrix R and the variance covariance matrix H_t as follows:

$$R = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{12} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1N} & \rho_{2N} & \cdots & 1 \end{bmatrix} \tag{6}$$

and

$$H_t = D_t R D_t \tag{7}$$

where $D_t = \mathrm{diag}(\sigma_{1t}, \sigma_{2t}, \cdots, \sigma_{Nt})$.

The basic idea is that every variance–covariance matrix can be decomposed in the above way. Therefore, we can characterize the dynamics in the following way.

$$H_t = \begin{bmatrix} \sigma_{1t}^2 & \sigma_{12,t} & \cdots & \sigma_{1N,t} \\ \sigma_{12,t} & \sigma_{2t}^2 & \cdots & \sigma_{2N,t} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N,t} & \sigma_{2N,t} & \cdots & \sigma_{Nt}^2 \end{bmatrix} \tag{8}$$

$$\sigma_{it}^2 = \omega_i + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 \quad i = 1, \cdots, n \tag{9}$$

$$\sigma_{ij,t} = \rho_{ij}\sigma_{it}\sigma_{jt} \quad i, j = 1, \cdots, n, i \neq j \tag{10}$$

The usual conditions to ensure the positivity of the variances and the stationarity hold : $\omega_i > 0$,$\alpha_{i,j} > 0$, $\beta_{i,j} > 0$ and $\sum_{j=1}^{q} \alpha_{i,j} + \sum_{j=1}^{p} \beta_{i,j} < 1$. [5] The total number of parameters is $(p + q + 1)N + \frac{N(N-1)}{2}$, when $p = q = 1$, $N = 2$, 7 parameters need to be estimated, which is not so many but still lack parsimony. Positive definiteness of the variance covariance matrix is controlled by the correlation matrix, while only the usual requirements of positivity constraints for GARCH model suffice. In order to abtain the parameters, maximum likelihood estimation method can be used.

A useful extension for the CCC model is the inclusion of a $K \times 1$ vector of exogenous variables X_t in order to incorporate the possible factors other than the investigated return series. We call it CCCX. To better capture the effects of exogenous variables imposed on the dynamics of correlation, we need to separate the common effect with the individual effect. Let $X_t = \{X_{1t}, X_{2t}, \cdots, X_{Kt}\}'$, where X_{it} is the representative exogenous variable. The CCCX model with all the exogenous elements in X_t playing an equal common effect among all the volatility dynamics (we refer to it

[5] For more rigorous conditions, see the Appendix.

as CCCXC) is formulated as follows:

$$\sigma_{it}^2 = \omega_i + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 + \sum_{l_1=1}^{m} \lambda_{1,l_1} X_{1,t-l_1} + \cdots + \sum_{l_K=1}^{r} \lambda_{K,l_K} X_{K,t-l_K} \quad (11)$$

for $i = 1, \cdots, N$. We define $\lambda_i = \{\lambda_{i1}, \cdots, \lambda_{ij}\}'$,for $i = 1, \cdots, K$ and $j = m, \cdots, r$, as the common effect with respect to every exogenous variable X_{it} and its lagged terms.

Analogously, we model the CCCX model with exogenous variables X_t exerting different individual effects according to the individual volatility dynamics (we refer to it as CCCXI) in the following way:

$$\sigma_{it}^2 = \omega_i + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 + \sum_{l_1=1}^{m} \lambda_{1,l_1}^i X_{1,t-l_1} + \cdots + \sum_{l_K=1}^{r} \lambda_{K,l_K}^i X_{K,t-l_K} \quad (12)$$

for $i = 1, \cdots, N$. Here,$\lambda_j^i = \{\lambda_{j1}^i, \cdots, \lambda_{jl}^i\}'$, for $i = 1, \cdots, n$, $j = 1, \cdots, K$ and $l = m, \cdots, r$, is the individual effect with respect to every exogenous variable X_{it} and its lagged terms.

Though CCC model has attracted us for its easiness of estimation and guarantee of positive definiteness of variance covariance matrix H_t, it faces one major problem. Constant correlation seems to be a too strong assumption most of the time. Empirical results tend to easily reject the assumption that correlation (both unconditional and conditional) is constant for most of the markets and assets. For instance, Tsui and Yu (1999) have used Chinese stock market data to test the validity of CCC assumption. They find that the null hypothesis of a constant condtional correlation in the stock returns cannot be supported. However, CCC model still remains to be a benchmark model when coming to the modeling the correlation. It serves as a good comparison.

3.3.2 DCC–GARCH Model

Considering the fact that constant conditional correlations over time is not realistic, researchers seek to generalize Bollerslev's CCC model. Followed by dynamic conditional correlation (DCC) model of Engle (2002) and time–varying correlation model of Tse and Tsui (2002), the time–varying features of correlations have been intensively examined for many markets and many classes of assets. The challenge is how to transform the constant correlation matrix R to its time–varying counterpart R_t. Engle's dynamic correlation strucuture is defined as follows:

$$H_t = D_t R_t D_t \tag{13}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{14}$$

$$Q_t = (1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n)\bar{Q} + \sum_{m=1}^{M} \alpha_m (u_{t-m} u_{t-m}^{'}) + \sum_{n=1}^{N} \beta_n Q_{t-n} \tag{15}$$

where $\bar{Q} = E[u_t u_t^{'}]$, α_m and β_n are scalars such that $\sum_{m=1}^{M} \alpha_m + \sum_{n=1}^{N} \beta_n < 1$. $u_t \sim U(0, R_t)$ is a $N \times 1$ vector of residuals standardized by their conditional standard deviations with typical element $u_{it} = \frac{\varepsilon_{it}}{\sqrt{h_{ii,t}}}$ which can be obtained when the univariate GARCH volatility models are estimated. Q_t^* is as follows:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{q_{22,t}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \cdots & \sqrt{q_{NN,t}} \end{bmatrix}$$

so that $Q_t^* = [q_{ii,t}^*] = [\sqrt{q_{ii,t}}]$ is a diagonal matrix with the square root of the ith diagonal element of Q_t on its ith diagonal position. The typical element of R_t will be of the form $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}$. Engle and Sheppard (2001) established that the positive definiteness of Q_t will necessarily and sufficiently ensure the positive definiteness of

R_t, which validify R_t as a correlation matrix. They use the unconditional variance–covariance matrix of the standardized residuals to replace the matrix $\bar{Q}$ when estimating the parameters, which is in line with the standard univariate GARCH results. That is, sample variance–covariance matrix $\hat{\bar{Q}} = \frac{\sum_{t=1}^{T} u_t u_t'}{T}$ serves as the estimator of $\bar{Q}$. This simplification invokes the concept of variance targeting introduced by Engle and Mezrich (1996). Variance targeting assumes that in the long run the process of Q_t will approach the sample variance–covariance matrix $\hat{\bar{Q}}$. Though variance targeting is achieved in this context, we cannot guarantee the positive definiteness of the variance–covariance matrix H_t. Hafner and Franses (2003) proposed a generalized DCC model to ensure the positive definiteness of the H_t matrix while sacrificing the variance targeting. Whether to choose variance targeting or not depends on the complexity of the model estimation. However, we should not expect major difference in these two categories. Similarly, correlation targeting is imposed when necessary. DCC model with exogenous variables is straightforwardly extended as the CCC model does.

Two–step estimation procedure is applied when estimating the DCC model. The first step will be the estimation of the univariate GARCH model. Then the estimation results are used as input to estimate the correlation parameters in the second step. Engle and Sheppard (2001) proved that the two–step estimator is consistent. If we let the unknown innovation series ζ_t assume the multivariate normal distribution, we have the following maximum likelihood function. Even without normality assumption, the estimators can still achieve the Quasi–Maximum Likelihood Estimator (QMLE) properties.

$$f(r_t) = (2\pi)^{-\frac{n}{2}} |H_t^{-\frac{1}{2}}| e^{-\frac{1}{2} r_t' H_t^{-1} r_t} \tag{16}$$

$$r_t|\mathcal{F}_{t-1} \sim \text{i.i.d. } N(\mathbf{0}, H_t)$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(N\log(2\pi) + \log|H_t| + r_t^{'}H_t^{-1}r_t) \tag{17}$$

Tse and Tsui (2002)'s time–varying model for the conditional matrix is formulated differently. With the decompostion of the variance covariance matrix

$$H_t = D_t R_t D_t \tag{18}$$

and the conditional variances modeled as a univariate GARCH model

$$\sigma_{it}^2 = \omega_i + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 \quad i = 1, \cdots, n \tag{19}$$

they propose a form for the time–varying correlation matrix that resembles similarly to an ARMA process:

$$R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \Psi_{t-1} + \theta_2 R_{t-1} \tag{20}$$

where the parameters θ_1 and θ_2 are nonnegative and satisfy $\theta_1 + \theta_2 < 1$. R is symmetric positive definite $N \times N$ matrix with unit diagonal elements. Ψ_{t-1} is the $N \times N$ correlation matrix of ζ_τ, the innovation terms for $\tau = t - M, t - M - 1, \cdots, t - 1$, where the single elements are defined as follows:

$$\psi_{ij,t} = \frac{\sum_{m=1}^{M} u_{i,t-1}u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{i,t-m}^2)(\sum_{m=1}^{M} u_{j,t-m}^2)}}$$

and

$$\psi_{ii,t} = 1$$

where $u_{it} = \frac{\varepsilon_{it}}{\sqrt{h_{ii,t}}}$.

If $M = 1$, Ψ_{t-1} equals to the unit matrix. We can see R_t is the weighted average of long–run unconditional correlation, the conditional correlation from the last period and the shock from the last period. To ensure the positive definiteness of Ψ_{t-1} and therefore of R_t, we need $M \geq N$.

3.3.3 Block–DCC–GARCH Model

Block DCC (BDCC) model developed by Billo, Caporin and Gobbo (2003, BCG hereafter) is a generalization of the dynamic conditional correlation (DCC) multivariate GARCH model. Though Engle (2002) added to the CCC model of Bollerslev (1990) some form of time–varying dynamics in the correlations, specifically, introducing a GARCH–type structure, the dynamics is restrictive, constraining the dynamics to be equal for all the correlations. This specification causes problems: consider for example a stock market, with the assets grouped in homogeneous categories (energy, food, chemistry, etc) or think a model for geographical areas, we may assume different patterns of correlation inside the groups and betweent the groups. Following this direction, BCG extended the DCC model to allow a block-diagonal structure that loose this restriction, in which the dynamics is constrained to be equal only among the specific groups of variables. The Block–DCC–GARCH model is obtained by reformulating the dynamic correlation equation in the following way:

$$Q_t = [I - \alpha(L) - \beta(L)] \odot \bar{Q} + \alpha(L) \odot \varepsilon_t \varepsilon_t^{'} + \beta(L) \odot Q_t \tag{21}$$

$$\alpha(L) = \sum_{i=1}^{\bar{q}} \alpha_i L^i, \quad \beta(L) = \sum_{j=1}^{\bar{p}} \beta_j L^j \tag{22}$$

where L^i is the time lag operator of order i, α_i and β_j are $N \times N$ matrices, $\odot$ indicates the Hadamard product and $\bar{Q} = E[u_t u_t']$ with sample equivalent $\hat{\bar{Q}} = \frac{\sum_{t=1}^{T} u_t u_t^{'}}{T}$ that serves as the estimator of $\bar{Q}$. The parameter matrices α_i and β_j are assumed with the following structure: when the N asset return series are grouped in w sets of dimension

m_1, m_2, $\cdots$, m_w such that $\sum_{k=1}^{w} m_k = w$, then

$$\alpha_i = \begin{bmatrix} \alpha_{i,11} i(m_1)i(m_1)' & \alpha_{i,12} i(m_1)i(m_2)' & \cdots & \alpha_{i,1w} i(m_1)i(m_w)' \\ \alpha_{i,21} i(m_2)i(m_1)' & \alpha_{i,22} i(m_2)i(m_2)' & \cdots & \alpha_{i,2w} i(m_2)i(m_w)' \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{i,w1} i(m_w)i(m_1)' & \alpha_{i,w2} i(m_w)i(m_2)' & \cdots & \alpha_{i,ww} i(m_w)i(m_w)' \end{bmatrix}$$

$$\beta_j = \begin{bmatrix} \beta_{j,11} i(m_1)i(m_1)' & \beta_{j,12} i(m_1)i(m_2)' & \cdots & \beta_{j,1w} i(m_1)i(m_w)' \\ \beta_{j,21} i(m_2)i(m_1)' & \beta_{j,22} i(m_2)i(m_2)' & \cdots & \beta_{j,2w} i(m_2)i(m_w)' \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{j,w1} i(m_w)i(m_1)' & \beta_{j,w2} i(m_w)i(m_2)' & \cdots & \beta_{j,ww} i(m_w)i(m_w)' \end{bmatrix}$$

where $i(m_k)$ is a column vector of ones with dimension m_k, $k = 1, 2, \cdots, w$. Positive definiteness is guaranteed as long as $I - \alpha(L) - \beta(L)] \odot \bar{Q}$ and $\varepsilon_t \varepsilon_t^{'}$ are positive definite following Engle and Sheppard (2001), which is not difficult to impose in this case.

The modeling asymmetric comovements of asset returns and correlations will substantially improve our understanding of market linkage and information absorption mechanisms. Ang and Chen (2002) finded that correlations of equity portfolios are asymmetric. McAleer, Chan, and Marinova (2002) analyzed asymmetric behaviors in patent markets. See Kroner and Ng (1998) for detailed information. The asymmetric generalized DCC–GARCH model (AGDCC) proposed by Cappiello, Engle, and Sheppard (2006, CES hereafter) incorporates the well known stylized facts in financial data—asymmetric effect to better explain the skewness. CES (2006) formulated their correlation structure as follows:

$$Q_t = [\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G] + A'\varepsilon_{t-1}\varepsilon_{t-1}^{'}A + B'Q_{t-1}B + G'n_{t-1}n_{t-1}^{'} \quad (23)$$

where A, B and G are $N \times N$ parameter matrices, $n_t = I[u_t < \tau] \odot u_t$ is the $N \times 1$ vector of asymmetric components (τ is a threshold of asymmetry that is typically set to zero, $I[\cdot]$ is a $N \times 1$ indicator function which takes on value 1 if the argument is true and 0 otherwise, $\odot$ is the Hadamard product as usual.) and $\bar{N} = E[n_t n_t^{'}]$ with sample equivalent $\hat{\bar{N}} = \frac{\sum_{t=1}^{T} n_t n_t^{'}}{T}$ that serves as the estimator of $\bar{N}$.

Considering the prevalent asymmetric effects in various financial data and also the possible block structures in the multivariate analysis, Vargas (2006) combined the models of BCG (2003) and CES (2006) and proposed the asymmetric block DCC–GARCH model (ABDCC) model. The specification is as follows:

$$\begin{aligned} Q_t &= [\bar{Q} - \alpha(L) \odot \bar{Q} - \beta(L) \odot \bar{Q} - \eta(L) \odot \bar{N}] + \alpha(L) \odot \varepsilon_t \varepsilon_t^{'} \\ &\quad + \beta(L) \odot Q_t + \eta(L) \odot (n_t n_t^{'}) \end{aligned} \tag{24}$$

$$\alpha(L) = \sum_{i=1}^{\bar{q}} \alpha_i L^i, \quad \beta(L) = \sum_{i=1}^{\bar{p}} \beta_i L^i, \quad \eta(L) = \sum_{k=1}^{\bar{h}} \eta_k L^k$$

where the parameter matrices of α_i and β_j are the same as above, and η_k is defined similarly as follows:

$$\eta_k = \begin{bmatrix} \eta_{k,11} i(m_1) i(m_1)' & \eta_{k,12} i(m_1) i(m_2)' & \cdots & \eta_{k,1w} i(m_1) i(m_w)' \\ \eta_{k,21} i(m_2) i(m_1)' & \eta_{k,22} i(m_2) i(m_2)' & \cdots & \eta_{k,2w} i(m_2) i(m_w)' \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{k,w1} i(m_w) i(m_1)' & \eta_{k,w2} i(m_w) i(m_2)' & \cdots & \eta_{k,ww} i(m_w) i(m_w)' \end{bmatrix}$$

Positive definiteness is obtained as above whenever $n_t n_t'$ and $\bar{Q} - \alpha(L) \odot \bar{Q} - \beta(L) \odot \bar{Q} - \eta(L) \odot \bar{N}$ are positive definite.

4 Data

One important contribution in this paper is that it examines the four Greater China stock markets simultaneously. The argument is straightforward and easily understood as these four markets have been historically closely interlinked due to cultural similarities and geographical closeness. Researchers have seldom investigated the four markets together. We use daily returns on stock indices for four Greater China region stock markets (Shanghai, Shenzhen, Hongkong and Singapore) plus an exogenous stock market index. Specifically, the stock indices are Shanghai Stock Exchange Composite Index (Shanghai), Shenzhen Stock Exchange Composite Index (Shenzhen), Hang Seng Index (Hongkong), and Straits Times Index (Singapore) respectively. Nikkei 225 Index (Japan) of Tokyo Stock Exchange is included in this paper as a representative exogenous index. [6] All the indices are price–weighted series of all listed stocks in the exchange. Indices are obtained from the Datastream International and are converted into daily return series without incorporating the effect of exchange rate fluctuation as our analysis focus on local currency returns. The sampling period for the five daily stock market indices is from January the 5th of 2000 to March the 19th of 2008, excluding all the public holidays and non–traded days for all the five stock markets simultaneously. There are 1712 observations in total for each return series. The return series r_{it} is formed by transforming the gross index into continuously compound rate of return (in percentages) from time $t-1$ to t by applying the common formula: $r_{it} = 100 \cdot (\log(P_{it}) - \log(P_{i,t-1}))$, where P_{it} is the value of index i at time t, for $i =$ Shanghai, Shenzhen, Hongkong, Singapore, Japan.

We start by analyzing the dynamic behavior of each univariate series, which serves to facilitate the multivariate modeling and the understanding of multivariate dynam-

[6] Which exogenous variable should we choose is subject to several considerations, among them we think same time zone, geographical influence, and economical importance are the three key factors.

ics. Figure 1 and Figure 2 present the five daily indices and corresponding return series of the sampling period. Table 1 reports the summary statistics. It's noted that both the Shanghai and Shenzhen markets experience positive mean returns, with Shanghai enjoying a greater average gain of 0.0267% than 0.0246% of Shenzhen. Based on the magnitude of the unconditional standard deviations, the Shenzhen market is more volatile. Both markets generate milder negative skewness and very high kurtosis, while the Shanghai market has both of them bigger in magnitude. Comparatively, Hongkong, Singapore, and Japan all present negative mean returns. The Hongkong market is the most volatile among the three in terms of unconditional standard deviations. Both Hongkong and Singapore generate larger skewness and kurtosis than their mainland China counterparts, while Hongkong has the largest skewness and kurtosis in magnitude. Japan stands out as a special case with a much larger loss of 0.0649% and a smaller skewness and kurtosis. The JB test statistics lead to a rejection of the assumption of normality of returns for all the five markets. Test statistics for the serial correlation in r_{it}, $|r_{it}|$, and r_{it}^2 are also reported in Table 1. The Ljung–Box's Q–statistics show that there are significant serial correlations in $|r_{it}|$ and r_{it}^2 for all the five stock markets. But serial correlation in r_{it} is not significant for Shenzhen and Japan markets at the 5% significance level. These results indicate that the return series of all markets exhibits conditional heteroskedastisity and that a GARCH process is an appealing candidate for modeling their time–series behavior.

5 Major Empirical Results

5.1 Econometric Interpretation

We shall focus on the simple models of order (1,1) in the subsequent empirical analysis, which is typically the case in financial applications and we can easily generalize these models to higher–order. All the subscripts involving the estimated coefficients below are assigned as follows: if the coefficient is a_{ij}, for $i, j = 1, 2, 3, 4$, then Shanghai $= 1$, Shenzhen $= 2$, Hongkong $= 3$ and Singapore $= 4$.

The estimated results for Diagonal VEC model and Diagonal BEKK model under multivariate normal distribution are presented in Tables 2, 3. For the diagonal VEC model, all the parameters involving the dynamics of variances are significant at the 1% significance level, while the parameters of covariances are significant except ω_{14} and ω_{24}, where ω_{14} and ω_{24} denote the intercept terms for dynamics of covariances between Shanghai and Singapore, Shenzhen and Singapore, respectively. Strong time–varying conditional volatility is present in all the four return series. Significant own–volatility spillovers [7] exist in all the four markets. The Diagonal BEKK model is simplified and easy to guarantee positive definiteness, but all of the intercept parameters are not significant except Shanghai and Shenzhen. In contrast, the parameters in related to lagged terms of innovations and own variances and covariances are all significant. In order to see the dynamics of variances and covariances, we need to expand the matrix and arrange the corresponding dynamics.

The conditional correlation plots drawn from Diagonal VEC and Diagonal BEKK models are only slightly differnt and identify the existance of an apparent upward trends in the correlations of Shanghai and Hongkong, Shanghai and Singapore, Shenzhen and Hongkong, and Shenzhen and Singapore, indicating the gradual integration

[7] "Own–volatility spillovers" is used to indicate a one–way causal relationship between past volatility shocks and current volatility in the same market. "Cross–volatility spillovers" is used to indicate a one–way causal relationship between past volatility shocks in one market and current volatility in another market.

of Chinese markets to the Global markets and the more closedness of the Great China region stock markets. However, it seems that the correlation between Shanghai and Shenzhen Markets is decreasing in recent years, though still staying a very high level, ranging from 0.55 to 0.97, nearly perfectly linear. On the other hand, Hongkong and Singapore are still integrating and forming an upward trend through the years, showing their more and more interdependence between each other. It should be noted that there are some striking breaks during the years, which either drag the correlations drastically low level in a very short time span or drive the correlations to some higher points, though the former movements are more obvious. We shall still have to investigate deeper to find the possible explanations to the sudden drop or up.

CCC models under multivariate normal distribution and multivariate Student's t distribution are estimated separately, attempting to partially explain the obvious non–normality due to negative skewness and high kurtosis in all the four return series. Results are shown in Tables 4 and 5. All parameters in the models are significant at the 1% significance level except the two intercept terms of Shanghai and Shenzhen in the first model. As expected, the coefficient of degree of freedom is much bigger than 2 and is significant. Furthermore, the loglikelihood increases a lot. All these evidences are a justification of improperness of assumption of normality for the return series. The constant conditional correlations through the sampling period between Shanghai and Shenzhen are 0.9403, Shanghai and Hongkong 0.1832, Shanghai and Singapore 0.1169, Shenzhen and Hongkong 0.1505, Shenzhen and Singapore 0.0929, and Hongkong and Singapore 0.6614. The numbers are average level and a very rough indication of the relatively integration level between the stock markets. We can draw some conclusions from the CCC model that are in accordance with the Diagnoal VEC model and Diagonal BEKK model. Nevertheless, we cannot see the dynamic paths of correlation between the four markets. Thus the constant conditional correlation assmption seems to be too restrictive. Estimation outcomes for CCC model with exogenous variable is presented in the Appendix.[8]

DCC model and models with exogenous variable are estimated with Nikkei 225 index as the exogenous variable. Both the common effect and the individual effect in this case are explored, where the common effect is taken as the single parameter entering the Q_t process while the individual effect is specified as a diagonal matrix with λ as its different individual effects in the Q_t process. Table 7, 8 and 9 show the estimation results. Significant time–varying correlations are discovered in all the DCC models as expected. DCCX models show that the Japanese stock market as exogenous variable exerts a positive effect towards the variances and covariances of the other four markets. That is, that Japanese stock market goes up will push the correlation of the other four market go up while if it goes down the correlation of the other four go down. This is not against common sense as Japan is the most powerful economic entity in Asia–Pacific region and investors put much more attention to Japanese capital markets. However, we notice that the conditional correlation plots are only slightly changed.

We only estimate the asymmetric block DCC models, which is in order to capture the well–known leverage effect in the data. Asymmetric DCC model with two different types of blocks are examined, specifically, ABDCC (2,2) and ABDCC (2,1,1), where ABDCC (2,2) is the model with Shanghai and Shenzhen as a block and Hongkong and Singapore as the other block; ABDCC (2,1,1) is the model with Shanghai and Shenzhen as a block, Hongkong and Singapore each as a block. The classification of blocks are easily justified, since Shanghai and Shenzhen share similar characteristics, for example, political environments, regulatory policies, type of shareholders and invetors, of mainland stock markets though still with a lot of other differences, Hongkong and Singapore as independent economic entities are similar in some point

[8] The small difference between the CCC, CCCXC and CCCXI model suggests that the assumption of constant conditional correlations is not sufficient to support influence of exogenous variabe. However, DCC models warrant us a closer look at where the actual difference between the correlations are.

and have differences as well. Block models show that the dynamics within each block is identical, but different across blocks, which help to detect the possible dynamics within and across blocks and therefore add much more flexibility. All the parameters are significant for ABDCC (2,2). Block structures and asymmetries in ABDCC (2,2) are justified by the coefficients α_i, β_i and η_i. All the β coefficients which indicate the own–correlation spillovers have shrunk to a samller scale with intercept terms growing much bigger in comparison with the non–block models. In this way, dynamics of correlation become much flatter than Diagonal VEC, Diagonal BEKK or DCC models. This finding also can be noted from the conditional correlation plots that the block model in some sense has diluted the correlation structure, since the conditional correlation series are not so volatile as the DCC ones do and the correlations seem to be packed up more tightly. We still can observe that the correlation between the block of Shanghai and Shenzhen and the stock market Hongkong or Singapore will have a upward trend. [9]

5.2 Empirical Findings

The above estimation results present the evidence of mainland stock markets's gradual integration to the global markets and the Greater China region markets are more and more integrated to each other. The results actually correspond to the many events we have encountered during the sampling period. Chinese stock markets have evolved a lot in the past several years and China's market capitalization of \$500 billion is increasingly attracting more foreign investors. Chinese markets have experienced major open–up policies in recent years. On 19 February 2001, Chinese government abandoned the restriction that B–share can only be traded by foreign investors and Chinese citizens are allowed to trade B–share in foreign currency. On 5 November

[9]Due to the large number of parameters needed to be estimated, the ABDCC (2,1,1) model does not converge, hence we don't include the estimation result for ABDCC (2,1,1). Even ignoring the non–covergence problem, we still have a large proportion of parameters that are insignificant, which is an indication of over–parameterization in the model. We have to face the trade–off between model flexibility and model complexity here.

2002, the China Securities Regulatory Commission (CSRC) and the People's Bank of China (PBOC) introduced the Qualified Foreign Institutional Investor (QFII) program as a provision for foreign capital to access China's financial markets. On 1 December 2002, the QFII act was enacted. Historically, A–share market in China was closed for foreign investors. Chinese QFII regulations relax some capital controls and allow foreign institutions to invest in RMB–denominated equity and bond markets. Indeed, QFII is a Chinese brokerage business, which allows qualified foreign institutions to trade Chinese A–share via special accounts opened at designated custodian banks, for their clients. The QFII mechanism not only further opens China's securities markets but also gives foreign investors an opportunity to take position on those markets and buy stakes in Chinese companies, thus sharing in China's phenomenal growth. QFIIs can provide their clients with added opportunities to share in the growth of the Chinese Market. As of the end of December 2007, a total of 52 foreign institutions have received QFII licenses and the market capitalization of shares held by them reached nearly RMB 200 billion, making them the main institutional investors in China's mainland capital markets. The introduction of QFII certainly makes Shanghai and Shenzhen markets more globalized and the correlation higher with other markets around the world.

On 13 April 2006, the Chinese government announced the Qualified Domestic Institutional Investor (QDII) scheme, allowing Chinese institutions and residents to entrust Chinese commercial banks to invest in financial products overseas. But the investment was limited to fixed-income and money market products. The Chinese government announced on 11 May 2007 to widen the scope of the QDII investment. In November 2007, Premier Wen Jiabao stated the need to further study the scheme for individual mainland Chinese residents to invest in stocks in Hong Kong. The QDII system has expanded the investment channels for domestic capital, enabling domestic investors to reasonably allocate their assets throughout the world and to reduce investment risks. Moreover, the system has directed the orderly outflow of capital and promoted an equilibrium in the balance of payments. Currently, all qualified commercial banks, insurance companies, fund companies, and securities companies can conduct QDII business, gradually diversifying the members of the QDII system. In particular, with the release of QDII products by fund companies in September 2007, the QDII business entered a phase of rapid development. The investment quota allowed to China's QDIIs reached \$42.17 billion by the end of September 2007. The QDII system will help stimulate the Chinese stock markets to be more integrated into the global capital markets. Due to the introdution of QFII and QDII, we can expect that the correlations between Chinese stock markets and Hongkong stock market, Singapore stock market will be increased and much more volatile as well.

The effect of introduction of QFII and QDII can be seen from Table 11, 12, and 13. We report the average correlations for the periods before QFII, between QFII and QDII, and after QDII. While the average correlation between QFII and QDII remains relatively stable comparing with the period before QFII, the correlations between Shanghai and Hongkong, Shanghai and Singapore, Shenzhen and Hongkong, Shenzhen and Singapore, and even Hongkong and Singapore experience an obvious increase during the period after QDII. These results are in line with our arguments above. More specifically, the highest average increase is observed for Shanghai–Singapore (0.0806 - 0.2059) followed by Shanghai–Hongkong (0.1460 - 0.2667), Shenzhen–Singapore (0.0677 - 0.1709), Shenzhen–Hongkong (0.1234 - 0.2154), and Hongkong–Singapore (0.6234 - 0.7079). It should be interesting to note that the correlation between Shanghai and Shenzhen is experiencing a mild decrease during the three periods.

5.2.1 Relationship between return, volatility and correlation

The tradeoff between return and correlation has long been an important topic, although there is no consensus of this topic. Table 14 present the correlation between the market return and the market correlation. All except the correlation between Hongkong–Singapore and Hongkong return are negative, which agrees with the findings of some researchers that low returns tend to be related to high correlation values.

Researchers are also very interested in the relationship between the volatility and correlation of assets or markets being investigated. We conduct a rough check about the four Greater China stock markets. Figure 16 and 17 present the scatter plots of the conditional correlation series against the volatility of the underlying markets.[10] The interesting feature we note is that for the correlations between Hongkong and Singapore, extreme volatility values are associated with high correlation values. This result agrees with the findings of other researchers, giving an indirect verification to our assertion that negative shocks cause higher volatilities and consequently higher correlations. However, this does not seem to hold for the mainland stock markets, where higher volatility values are not necessarily associated with high correlation values. There is an obvious clustering in Figure 15, where low volatility values correspond to high correlation values. The obvious contradictory finding for Chinese mainland stock markets remains to be an interesting topic in the future.

[10] All the plots are very similar either to Plot 1 or Plot 2, depending on whether it is the mainland stock market or otherwise.

6 Conclusion

In this paper we presented the basic and most common approaches in the class of multivariate generalized ARCH models and apply them to the Great China region stock market data. In the first part, the basic idea behind the MARCH theory and the basic problems were described.

In the second part of this paper we investigated the volatility spillover and correlation contagion among the four stock markets of Hongkong, Shanghai, Shenzhen and Singapore in the so–called Great China region. We find that the volatility spillover between Shanghai and Shenzhen is obvious and correlation contagion is detected. Conditional variance and conditional correlations are time varying and dynamic which conforms to the arguments in most of the literature. Shanghai and Shenzhen present a very high correlation level during the sampling period,varying from 0.75 to 0.98, at some point even near linear correlation, which is not uncommon due to the close interlink between the two markets. Hongkong and Singapore presents a mildly high correlation, varying from 0.25 to 0.9, with an average of 0.62. However, the correlation is very volatile. Results present the convincing evidence that Chinese stock markets are more and more integrated to the global markets and the Great China region markets are more integrated to each other. There are many obvious correlation breaks,when all the correlations suddenly drop to a drastically low level. The drop corresponds to the actual economic event as we discover. We also explore the relationship between return and correlation, and volatility and correlation.

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Technical preliminaries needed in the paper are summarized in Appendix A. Appendix B presents the tables and figures of data used and MARCH models estimated in the paper.

A Univariate GARCH Models

In this appendix, we describe the univariate GARCH specifications which we use in this paper.

1. GARCH:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

2. GARCH–M Model:

$$\begin{aligned}\mu_{it} &= m_i + \beta_{SH,i}r_{SH,t-1} + \beta_{SZ,i}r_{SZ,t-1} + \beta_{HK,i}r_{HK,t-1} + \beta_{SG,i}r_{SG,t-1} + \delta_i\sigma_{it} \\ &= m_i + r_{t-1}'\beta_i + \delta_i\sigma_{it}\end{aligned}$$

where m_i is the intercept; $r_{t-1} = [r_{SH,t-1}, r_{SZ,t-1}, r_{HK,t-1}, r_{SG,t-1}]'$ is a 4×1 column vector of past stock market returns.

$$\begin{aligned}\sigma_{it}^2 &= \omega_i + \alpha_{SH,i}\varepsilon_{SH,t-1}^2 + \alpha_{SZ,i}\varepsilon_{SZ,t-1}^2 + \alpha_{HK,i}\varepsilon_{HK,t-1}^2 + \alpha_{SG,i}\varepsilon_{SG,t-1}^2 + \gamma_i\sigma_{i,t-1}^2 \\ &= \sigma_i + \epsilon_{t-1}'\alpha_i + \gamma_i\sigma_{i,t-1}^2\end{aligned}$$

where $\epsilon_{t-1} = [\varepsilon_{SH,t-1}^2, \varepsilon_{SZ,t-1}^2, \varepsilon_{HK,t-1}^2, \varepsilon_{SG,t-1}^2]'$ is a 4×1 column vector of past squared innovations (i.e., $\varepsilon_{i,t-1}^2 = (r_{i,t-1} - \mu_{i,t-1})^2$).

3. Glosten-Jagannathan-Runkle GARCH (GJR–GARCH):

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I[\varepsilon_{t-1} < 0]\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Jeantheau(1998) showed that the log–moment regularity condition given by

$$E(\log(\alpha_1\eta_t^2 + \beta)) < 0$$

is sufficient for the QMLE to be consistent for the GARCH(1,1) model.

The second moment condition, namely $\alpha_1 + \frac{\gamma}{2} + \beta_1 < 1$, is sufficient for consistency and asymptotic normality of the QMLE for GJR (1,1) model. Moreover, McAleer, Chan and Marinova (2002) established the log–moment regularity condition for the GJR (1,1) model, namely

$$E(\log(\alpha_1 + \gamma_1 I(\eta_t)\eta_t^2 + \beta_1)) < 0$$

and showed that it is sufficient for the consistency and asymptotic normality of the QMLE for GJR (1,1) model.

B Tables and Figures

B.1 Tables

Table 1: Summary statistics of daily return series r_{it}

Statistics	Shanghai	Shenzhen	Hongkong	Singapore	Japan
Mean	0.0267	0.0246	-0.0202	-0.0265	-0.0649
Median	0.0444	0.0935	0.0269	0.0215	-0.0451
Maximum	8.8491	8.6669	6.5089	5.9422	5.7352
Minimum	-14.1681	-13.3855	-15.9720	-9.2155	-10.8903
Std.Dev.	1.6146	1.6873	1.4483	1.2067	1.4406
Skewness	-0.5938	-0.5270	-1.1713	-0.7564	-0.4527
Kurtosis	11.1756	8.4341	14.6208	9.2833	6.0671
JB	4868.5210	2185.6470	10024.6000	2979.4550	729.5178
P–value	0.0000	0.0000	0.0000	0.0000	0.0000
$Q(15)$	26.2280	18.8580	33.9260	33.6690	10.2890
P–value	0.0360	0.2200	0.0030	0.0040	0.8010
$\|Q\|(15)$	363.4500	446.8900	659.6000	535.7900	321.3100
P–value	0.0000	0.0000	0.0000	0.0000	0.0000
$Q^2(15)$	121.3100	122.7670	140.8300	146.1900	151.8400
P–value	0.0000	0.0000	0.0000	0.0000	0.0000

Cross–correlation

Markets	Shanghai	Shenzhen	Hongkong	Singapore	Japan
Shanghai	1	0.9203	0.2430	0.1574	0.1425
Shenzhen		1	0.1889	0.1201	0.1127
Hongkong			1	0.6837	0.6075
Singapore				1	0.5668
Japan					1

1. The Jarque-Bera test is a goodness–of–fit measure of departure from normality, based on the sample kurtosis and skewness. Under the null hypothesis of normality, the statistic JB has an asymptotic chi-square distribution with two degrees of freedom.

2. $Q(k)$, $|Q|(k)$, and $Q^2(k)$ are the Ljung–Box portmanteau test statistics for serial correlation of k lags of the original, absolute and squared return series, respectively. Under the null hypothesis of no serial correlation, the Q–statistics follows the chi-squared distribution with k degrees of freedom.

Table 2: Parameter estimates of Diagonal VEC under Multivariate Normal Distribution

$$\sigma_{it}^2 = \omega_{ii} + \alpha_{ii}\varepsilon_{i,t-1}^2 + \beta_{ii}\sigma_{i,t-1}^2$$
$$\sigma_{ij,t} = \omega_{ij} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_{ij}\sigma_{ij,t-1}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
ω_{11}	0.0558	0.0070	7.9944	0.0000
α_{11}	0.0882	0.0057	15.5162	0.0000
β_{11}	0.8974	0.0062	145.8146	0.0000
ω_{12}	0.0583	0.0075	7.7698	0.0000
α_{12}	0.0904	0.0058	15.6100	0.0000
β_{12}	0.8943	0.0063	142.9879	0.0000
ω_{13}	0.0066	0.0025	2.6169	0.0089
α_{13}	0.0275	0.0059	4.6470	0.0000
β_{13}	0.9441	0.0119	79.4864	0.0000
ω_{14}	0.0028	0.0019	1.4821	0.1383
α_{14}	0.0299	0.0073	4.0955	0.0000
β_{14}	0.9422	0.0136	69.4416	0.0000
ω_{22}	0.0618	0.0086	7.1726	0.0000
α_{22}	0.1002	0.0066	15.2504	0.0000
β_{22}	0.8881	0.0067	132.3494	0.0000
ω_{23}	0.0073	0.0032	2.3206	0.0263
α_{23}	0.0260	0.0063	4.0947	0.0000
β_{23}	0.9403	0.0156	60.2500	0.0000
ω_{24}	0.0030	0.0021	1.4261	0.1538
α_{24}	0.0293	0.0078	3.7415	0.0002
β_{24}	0.9366	0.0168	55.6693	0.0000
ω_{33}	0.0151	0.0034	4.4590	0.0000
α_{33}	0.0451	0.0053	8.4586	0.0000
β_{33}	0.9456	0.0059	160.0570	0.0000
ω_{34}	0.0149	0.0024	6.2979	0.0000
α_{34}	0.0532	0.0050	10.6549	0.0000
β_{34}	0.9299	0.0057	163.4090	0.0000
ω_{44}	0.0202	0.0030	6.7397	0.0000
α_{44}	0.0705	0.0065	10.9241	0.0000
β_{44}	0.9156	0.0064	143.6425	0.0000

Table 3: Parameter estimates of Diagonal BEKK under Multivariate Normal Distribution

$$H_t = \Omega\Omega' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}\text{ B'}$$

Parameter	Coefficient	P value		
α_1	0.2508	0.0056	44.5320	0.0000
α_2	0.2941	0.0070	41.9746	0.0000
α_3	0.1589	0.0077	20.6189	0.0000
α_4	0.2220	0.0104	21.2734	0.0000
β_1	0.9659	0.0013	750.6490	0.0000
β_2	0.9580	0.0020	485.0068	0.0000
β_3	0.9859	0.0015	644.0716	0.0000
β_4	0.9682	0.0029	339.5856	0.0000
ω_1	0.1892	0.0097	19.4358	0.0000
ω_2	0.1918	0.0138	13.8850	0.0000
ω_3	0.0119	0.0082	1.4430	0.1490
ω_4	0.0012	0.0099	0.1219	0.9030
ω_5	0.0056	0.0239	0.2324	0.8163
ω_6	-0.0257	0.1553	-0.1653	0.8687
ω_7	-0.0942	0.4115	-0.2289	0.8190
ω_8	0.0774	0.0529	1.4635	0.1433
ω_9	0.1071	0.2442	0.4388	0.6608
ω_{10}	-0.0024	5.1829	-0.0005	0.9996

where $\Omega = \begin{bmatrix} \omega_1 & 0 & 0 & 0 \\ \omega_2 & \omega_5 & 0 & 0 \\ \omega_3 & \omega_6 & \omega_8 & 0 \\ \omega_4 & \omega_7 & \omega_9 & \omega_{10} \end{bmatrix}$, $A = diag(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $B = diag(\beta_1, \beta_2, \beta_3, \beta_4)$.

Table 4: Parameter estimates of CCC under Multivariate Normal Distribution

$$\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$$
$$\sigma_{ij,t} = \rho_{ij} \sigma_{it} \sigma_{jt}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
ω_1	0.0037	0.0036	1.0475	0.2948
α_1	0.1142	0.0057	19.7864	0.0000
β_1	0.9043	0.0038	236.1939	0.0000
ω_2	0.0024	0.0034	0.7142	0.4751
α_2	0.1069	0.0060	17.6792	0.0000
β_2	0.9081	0.0039	230.2458	0.0000
ω_3	0.0155	0.0035	4.4024	0.0000
α_3	0.0408	0.0061	6.6487	0.0000
β_3	0.9502	0.0068	139.7198	0.0000
ω_4	0.0211	0.0029	7.3324	0.0000
α_4	0.0653	0.0064	10.2429	0.0000
β_4	0.9206	0.0064	144.7375	0.0000
ρ_{12}	0.9403	0.0020	469.0885	0.0000
ρ_{13}	0.1832	0.0209	8.7650	0.0000
ρ_{14}	0.1169	0.0226	5.1637	0.0000
ρ_{23}	0.1505	0.0226	6.6739	0.0000
ρ_{24}	0.0928	0.0232	3.9983	0.0001
ρ_{34}	0.6614	0.0120	55.2068	0.0000

Table 5: Parameter estimates of CCC under Multivariate Student's t Distribution

$$\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$$
$$\sigma_{ij,t} = \rho_{ij} \sigma_{it} \sigma_{jt}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
ω_1	0.0285	0.0074	3.8721	0.0001
α_1	0.0811	0.0084	9.6927	0.0000
β_1	0.9141	0.0077	118.4755	0.0000
ω_2	0.0213	0.0072	2.9600	0.0031
α_2	0.0872	0.0089	9.7489	0.0000
β_2	0.9163	0.0073	124.6950	0.0000
ω_3	0.0085	0.0029	2.9145	0.0036
α_3	0.0300	0.0052	5.7438	0.0000
β_3	0.9678	0.0049	196.2645	0.0000
ω_4	0.0121	0.0035	3.4053	0.0007
α_4	0.0468	0.0071	6.5742	0.0000
β_4	0.9484	0.0069	137.5174	0.0000
ν	5.3346	0.3160	16.8793	0.0000
ρ_{12}	0.9570	0.0022	444.8949	0.0000
ρ_{13}	0.1547	0.0288	5.3708	0.0000
ρ_{14}	0.1024	0.0290	3.5343	0.0004
ρ_{23}	0.1376	0.0290	4.7502	0.0000
ρ_{24}	0.0886	0.0286	3.0950	0.0020
ρ_{34}	0.6241	0.0163	38.2868	0.0000

Table 6: Parameter estimates of CCCXI under Multivariate Normal Distribution

Parameter	Coefficient	Standard Error	Test Statistic	P–value
ω_1	0.0064	0.0142	0.4500	0.6527
α_1	0.1138	0.0232	4.9140	0.0000
β_1	0.9024	0.0211	42.6962	0.0000
λ_1	-0.0213	0.0274	-0.7761	0.4377
ω_2	0.0035	0.0110	0.3194	0.7494
α_2	0.1059	0.0181	5.8368	0.0000
β_2	0.9077	0.0156	58.2363	0.0000
λ_2	-0.0207	0.0196	-1.0521	0.2927
ω_3	0.0188	0.0051	3.6712	0.0002
α_3	0.0374	0.0061	6.1376	0.0000
β_3	0.9503	0.0063	150.3841	0.0000
λ_3	-0.0246	0.0159	-1.5482	0.1216
ω_4	0.0224	0.0069	3.2662	0.0011
α_4	0.0458	0.0077	5.9744	0.0000
β_4	0.9333	0.0110	85.0203	0.0000
λ_4	-0.0463	0.0159	-2.9209	0.0035
ρ_{12}	0.9402	0.0062	152.0944	0.0000
ρ_{13}	0.1848	0.0371	4.9805	0.0000
ρ_{14}	0.1213	0.0335	3.6179	0.0003
ρ_{23}	0.1525	0.0284	5.3660	0.0000
ρ_{24}	0.0979	0.0278	3.5256	0.0004
ρ_{34}	0.6502	0.0193	33.6442	0.0000

Table 7: Parameter estimates of DCC under Multivariate Normal Distribution

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
α	0.0496	0.0002	260.4448	0.0000
β	0.9010	0.0007	1381.3440	0.0000

Table 8: Parameter estimates of DCCXC under Multivariate Normal Distribution

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1} + \lambda I_4 X_{t-1}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
α	0.0096	4.43E−06	2161.3710	0.0000
β	0.9004	6.13E−06	146858.5000	0.0000
λ	0.0099	4.69E−07	21136.7400	0.0000

Table 9: Parameter estimates of DCCXI under Multivariate Normal Distribution

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1} + diag(\lambda_1, \lambda_2, \lambda_3, \lambda_4) X_{t-1}$$

Parameter	Coefficient	Standard Error	Test Statistic	P–value
α	0.0093	3.71E−07	25153.0000	0.0000
β	0.9000	3.06E−07	2944955.0000	0.0000
λ_1	0.0174	9.43E−06	1844.2050	0.0000
λ_2	-0.0149	3.53E−05	-422.4441	0.0000
λ_3	0.0482	1.78E−06	27142.3100	0.0000
λ_4	-0.0845	0.0003	-254.0479	0.0000

Table 10: Parameter estimates of ABDCC(2,2) under Multivariate Normal Distribution

Parameter	Coefficient	Standard Error	Test Statistic	P–value
α_1	0.0996	1.37E−06	72649.4000	0.0000
β_1	0.8034	1.00E−06	801418.4000	0.0000
η_1	0.0444	1.16E−06	38192.8300	0.0000
α_2	0.1632	5.63E−05	2900.6290	0.0000
β_2	0.6949	0.0001	4428.3700	0.0000
η_2	-0.0522	9.81E−05	-531.7938	0.0000
α_3	0.0693	1.14E−05	6092.2380	0.0000
β_3	0.7336	5.73E−06	127971.1000	0.0000
η_3	0.0682	6.09E−06	11195.0700	0.0000

Table 11: Average correlation before QFII

	Shanghai	Shenzhen	Hongkong	Singapore
Shanghai	1	0.9485	0.1460	0.0806
Shenzhen		1	0.1234	0.0677
Hongkong			1	0.6234
Singapore				1

Average correlation is obtained by averaging the conditional correlation series produced from DCC estimation. The following 2 tables have the same structure.

Table 12: Average correlation between QFII and QDII

	Shanghai	Shenzhen	Hongkong	Singapore
Shanghai	1	0.9468	0.1555	0.0859
Shenzhen		1	0.1348	0.0676
Hongkong			1	0.5989
Singapore				1

Table 13: Average correlations after QDII

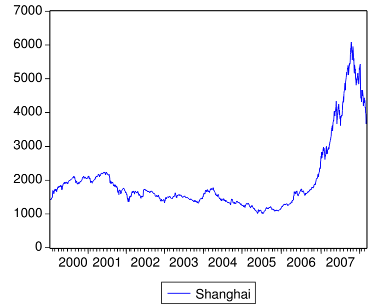
	Shanghai	Shenzhen	Hongkong	Singapore
Shanghai	1	0.9105	0.2667	0.2059
Shenzhen		1	0.2154	0.1709
Hongkong			1	0.7079
Singapore				1

Table 14: Correlation between return and market correlation

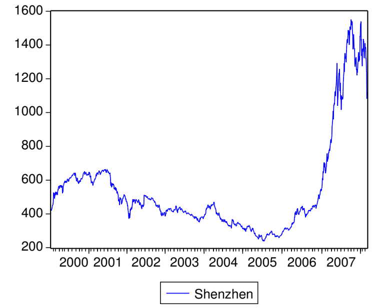
	Shanghai	Shenzhen	Hongkong	Singapore
Shanghai–Shenzhen	-0.0771	-0.0631		
Shanghai–Hongkong	-0.0045		-0.0290	
Shanghai–Singapore	-0.0675			-0.0252
Shenzhen–Hongkong		-0.0405	-0.0144	
Shenzhen–Singapore		-0.0811		-0.0235
Hongkong–Singapore			0.0117	-0.0537

The first column represents the correlation series obtained from DCC model. For example, Shanghai–Shenzhen is the conditional correlation series of Shanghai and Shenzhen markets. The first row is the return series.

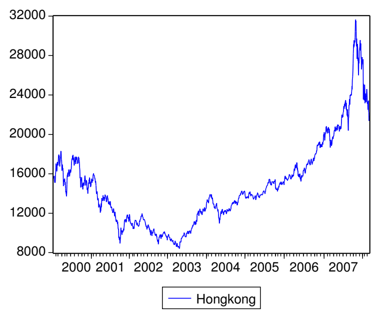
B.2 Figures



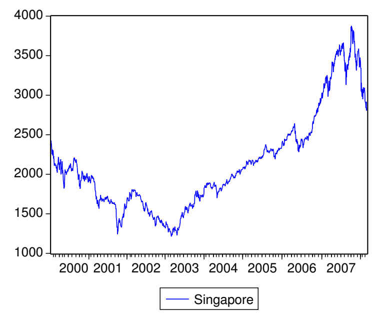
(a) Shanghai



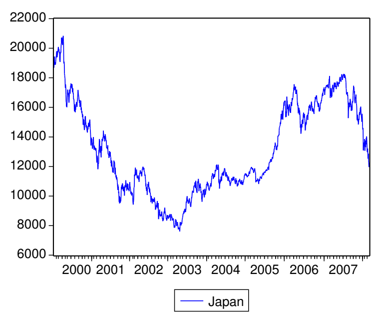
(b) Shenzhen



(c) Hongkong

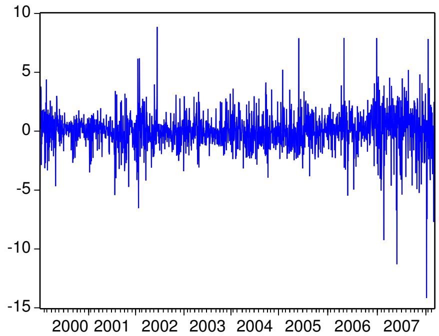


(d) Singapore

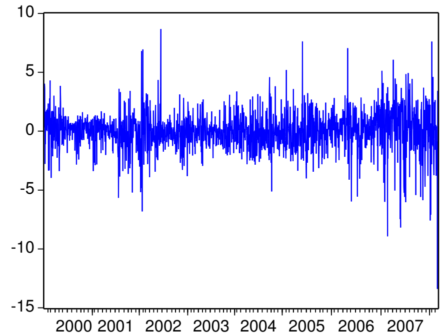


(e) Japan

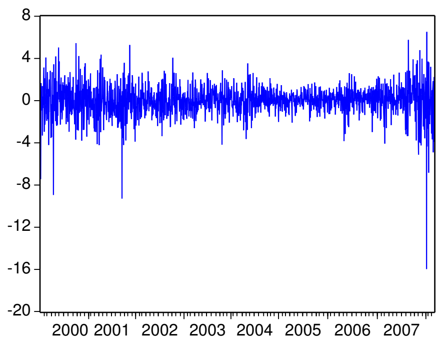
Figure 1: Daily index series P_{it}



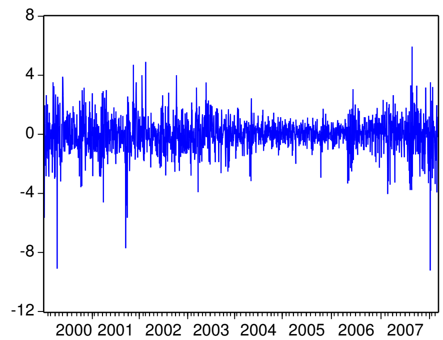
(a) Shanghai



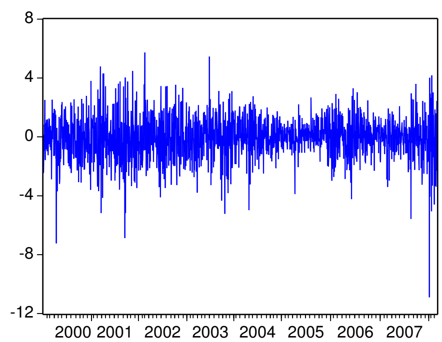
(b) Shenzhen



(c) Hongkong



(d) Singapore



(e) Japan

Figure 2: Daily return series r_{it}

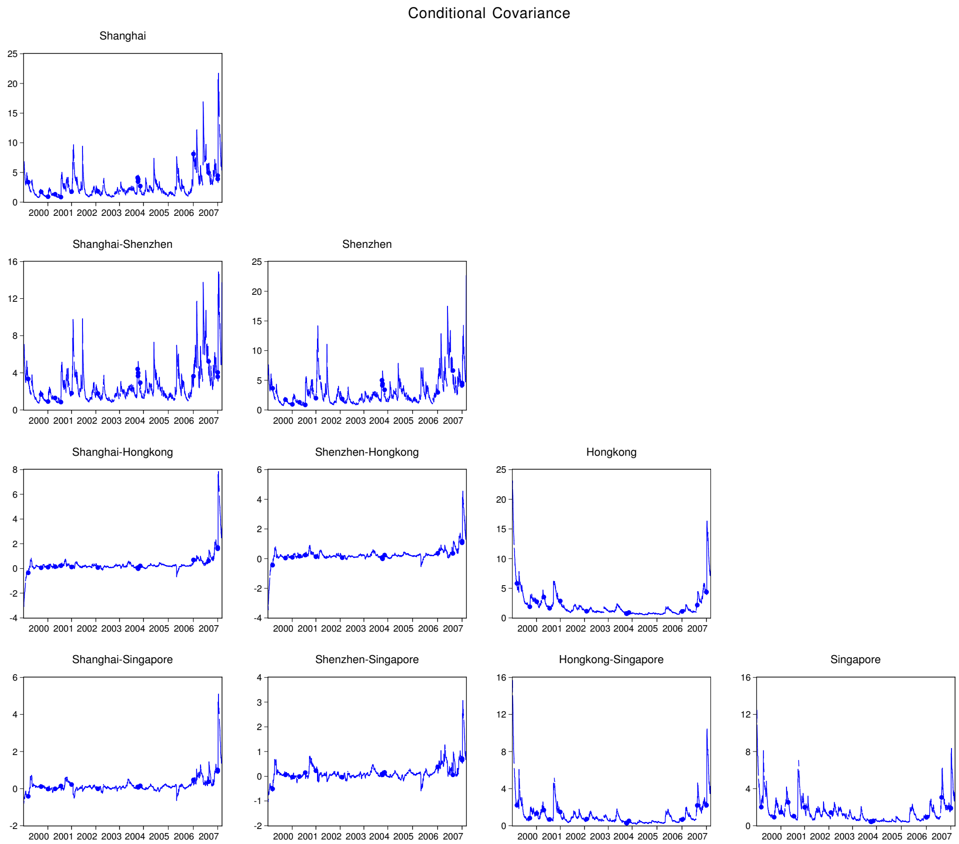


Figure 3: Variance–covariance of SH, SZ, HK and SG of Diagonal–VEC under Multivariate Normal Distribution

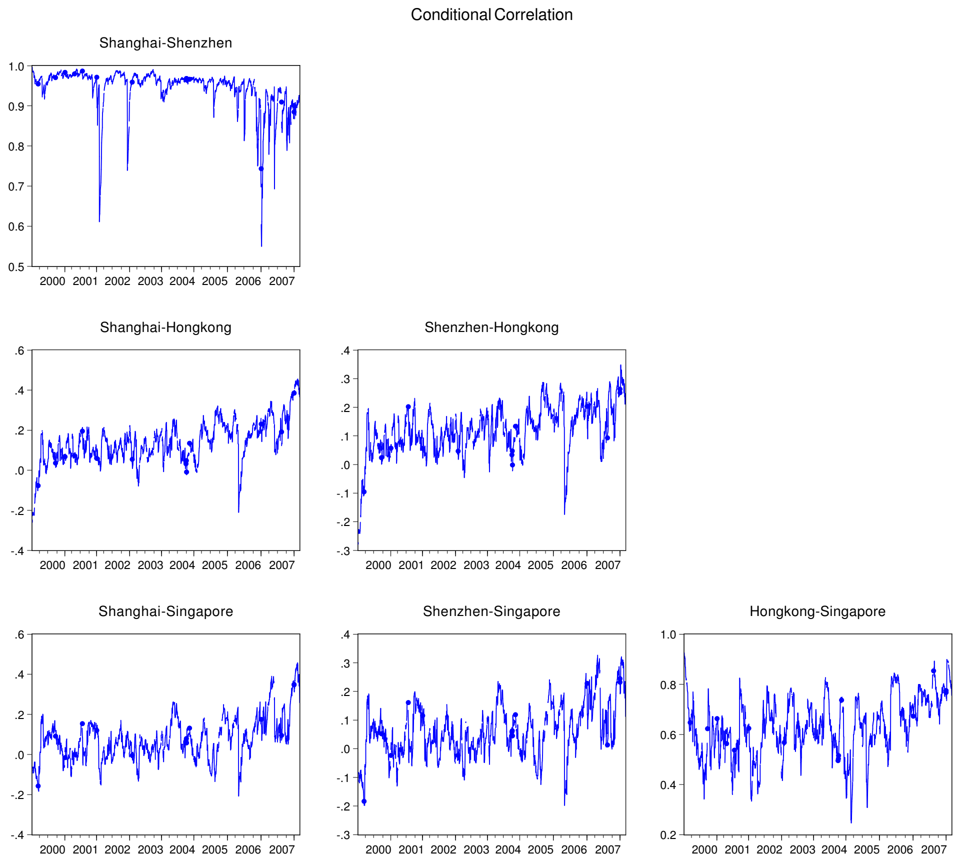


Figure 4: Correlation of SH, SZ, HK and SG of Diagonal–VEC under Multivariate Normal Distribution

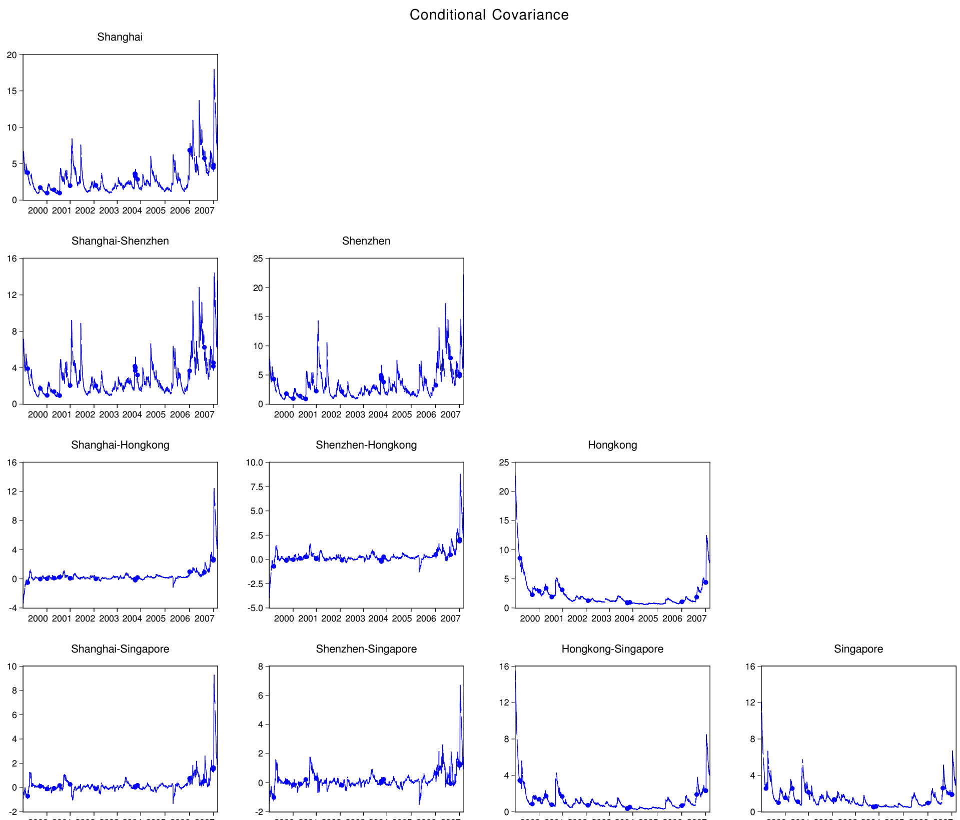


Figure 5: Variance–covariance of SH, SZ, HK and SG of Diagonal–BEKK under Multivariate Normal Distribution

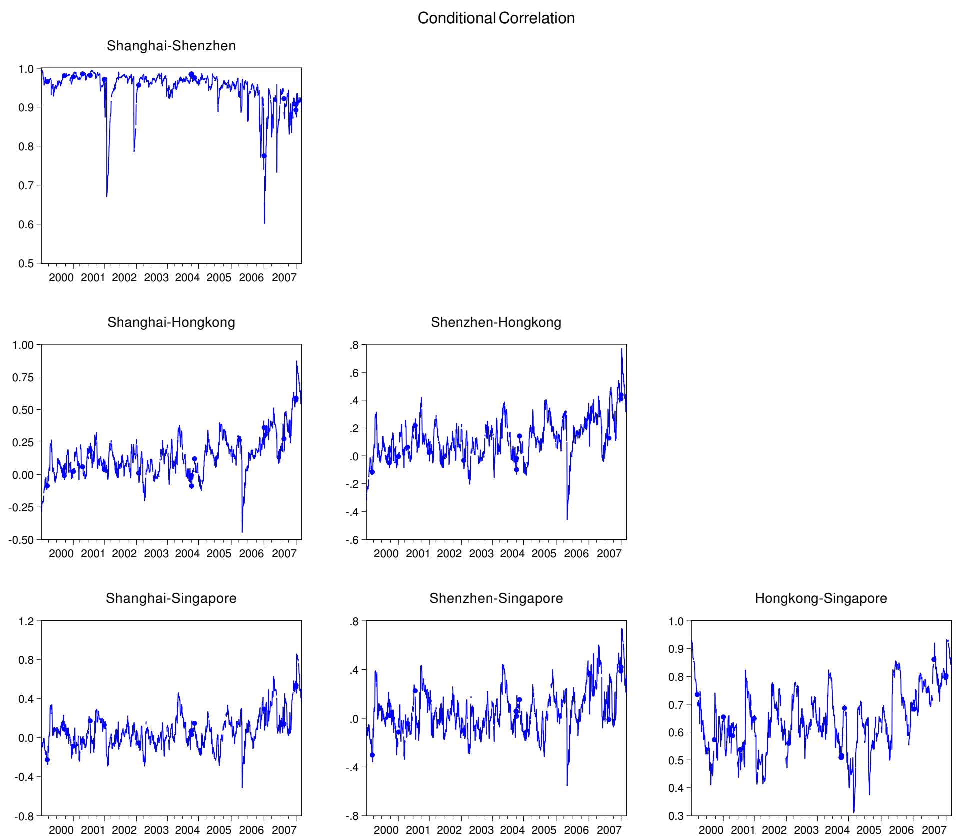


Figure 6: Correlation of SH, SZ, HK and SG of Diagonal–BEKK under Multivariate Normal Distribution

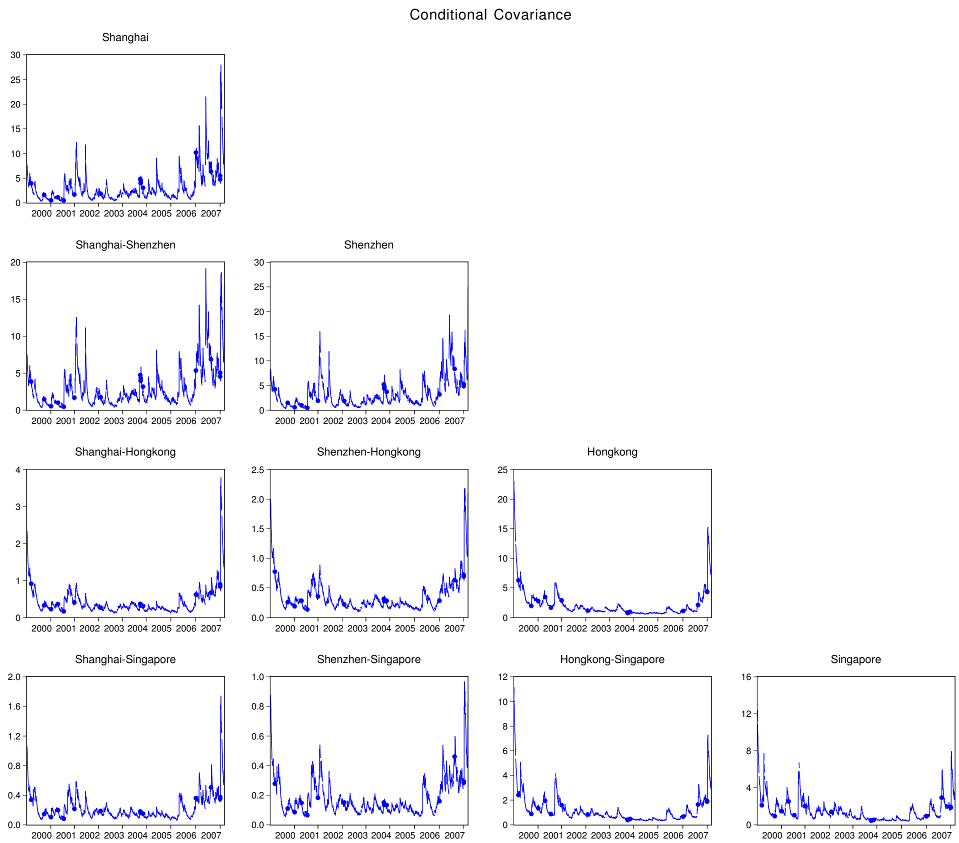


Figure 7: Variance of SH, SZ, HK and SG of CCC under Multivariate Normal Distribution

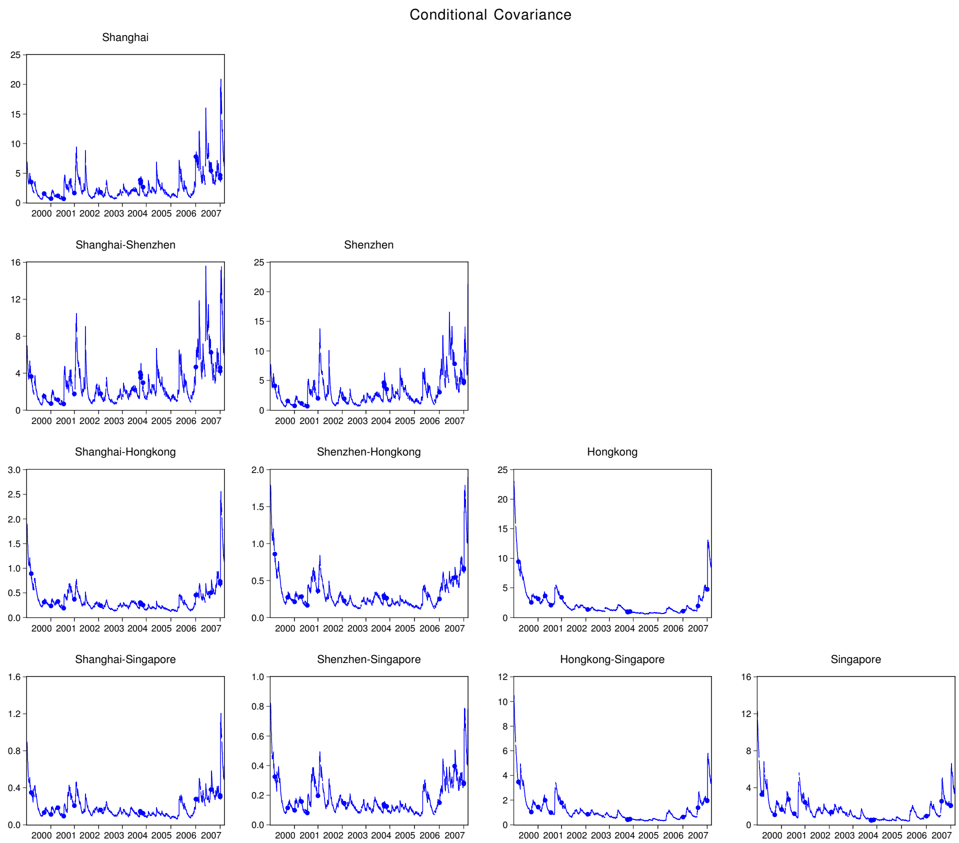


Figure 8: Variance of SH, SZ, HK and SG of CCC under Multivariate Student's t Distribution

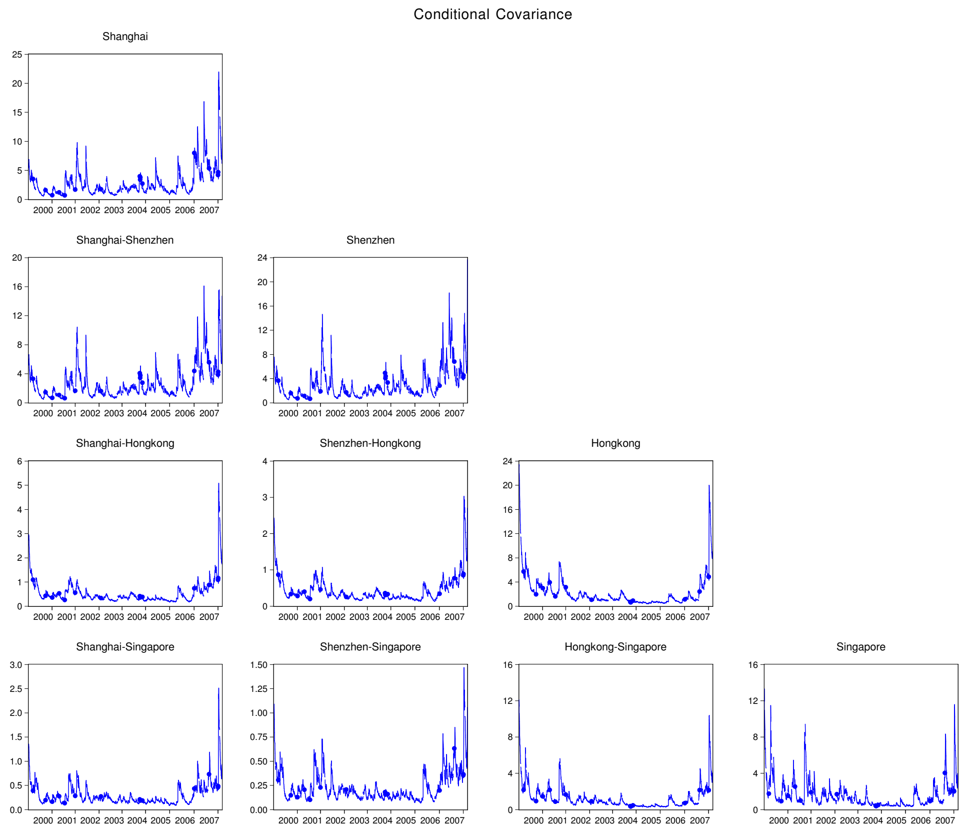


Figure 9: Variance of SH, SZ, HK and SG CCCXC under Multivariate Normal Distribution

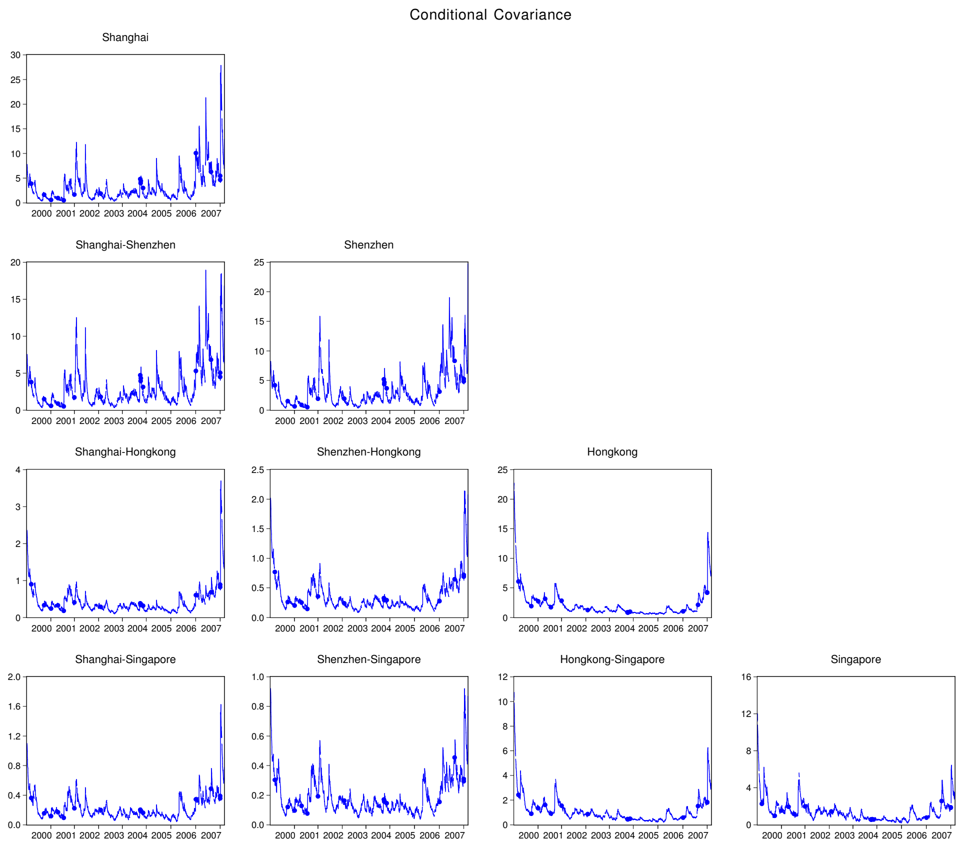
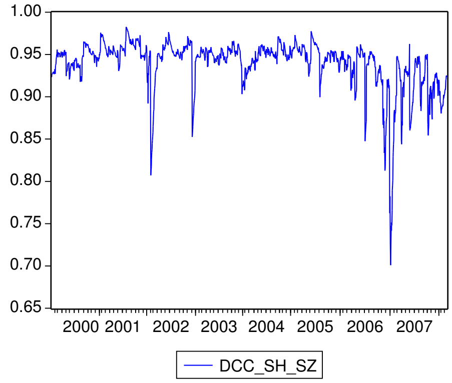
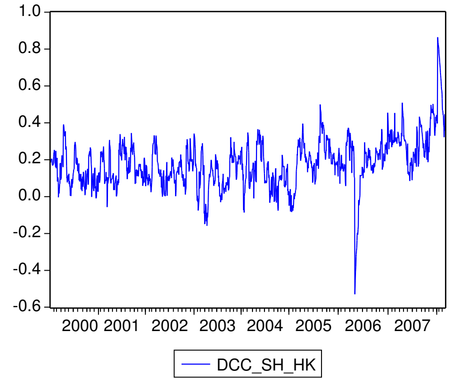


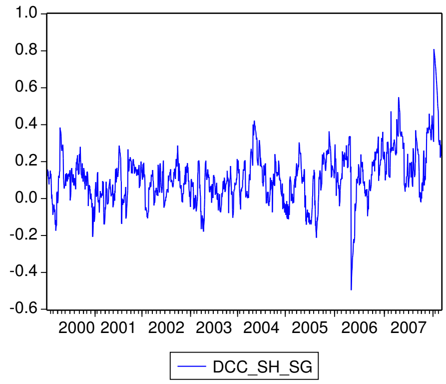
Figure 10: Variance of SH, SZ, HK and SG of CCCXI under Multivariate Normal Distribution



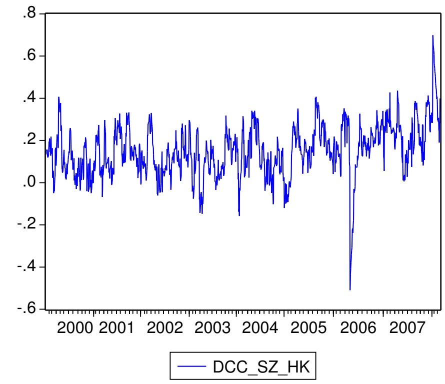
(a) Shanghai–Shenzhen



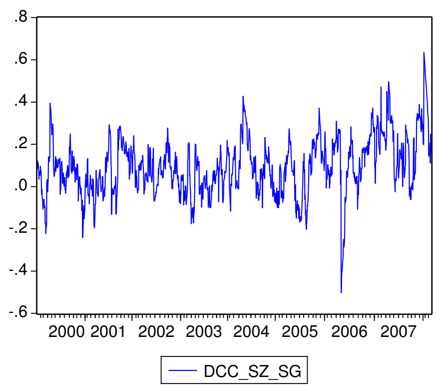
(b) Shanghai–Hongkong



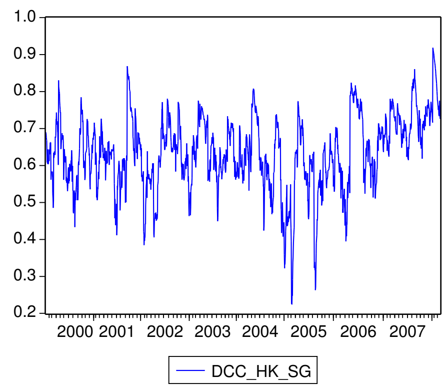
(c) Shanghai–Singapore



(d) Shenzhen–Hongkong

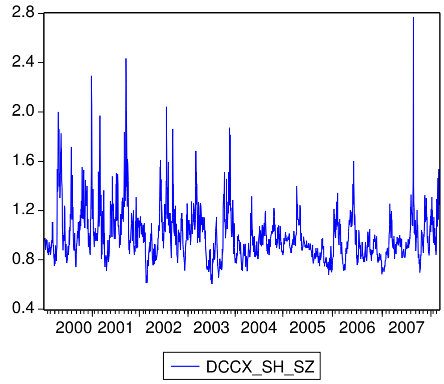


(e) Shenzhen–Singapore

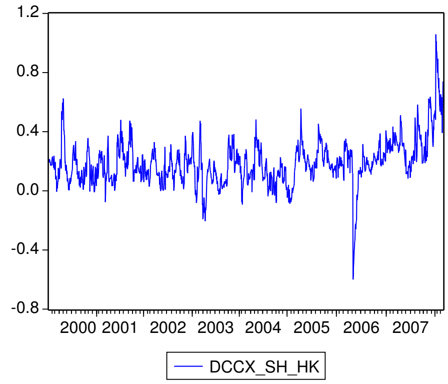


(f) Hongkong–Singapore

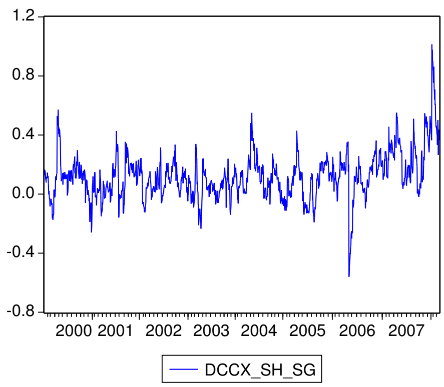
Figure 11: Conditional Correlation of DCC under Multivariate Normal Distribution



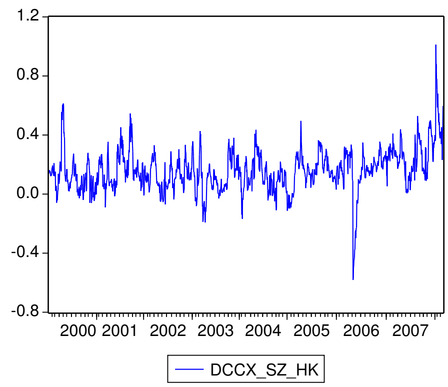
(a) Shanghai–Shenzhen



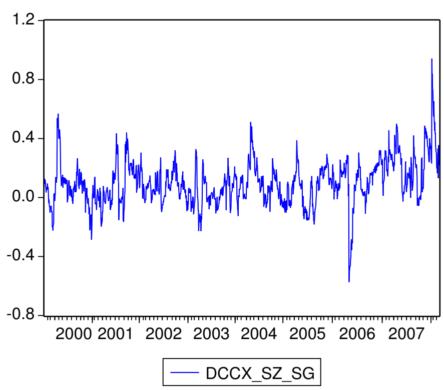
(b) Shanghai–Hongkong



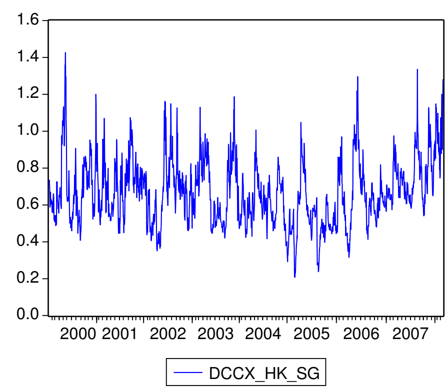
(c) Shanghai–Singapore



(d) Shenzhen–Hongkong

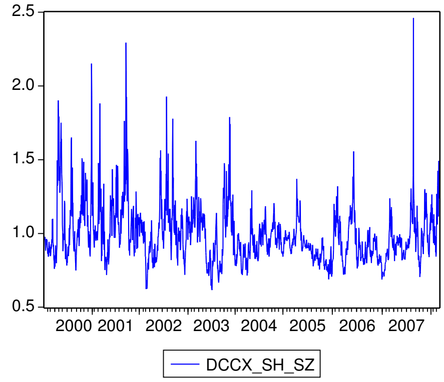


(e) Shenzhen–Singapore

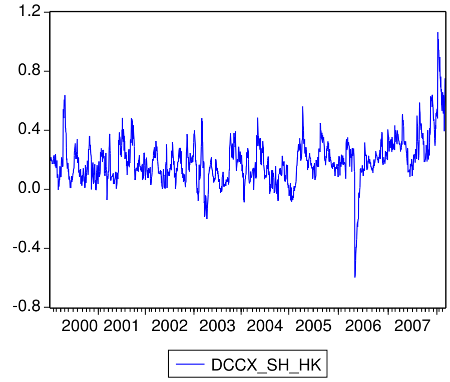


(f) Hongkong–Singapore

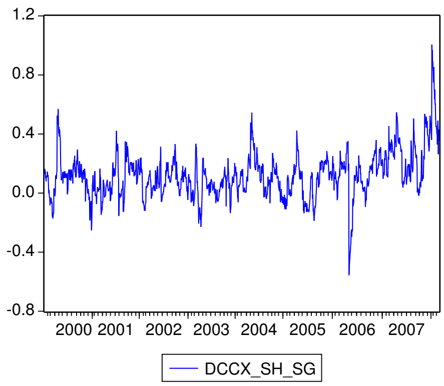
Figure 12: Conditional Correlation of DCCXC under Multivariate Normal Distribution



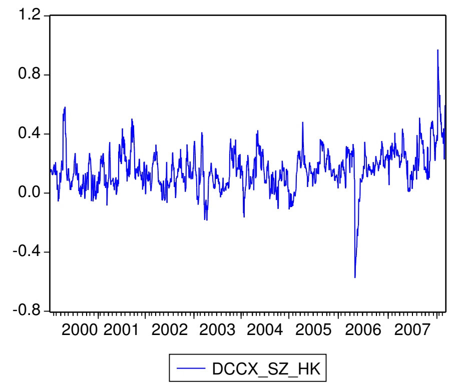
(a) Shanghai–Shenzhen



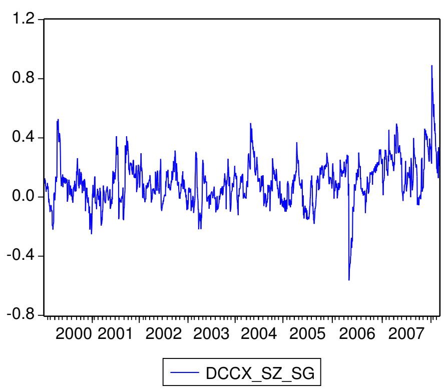
(b) Shanghai–Hongkong



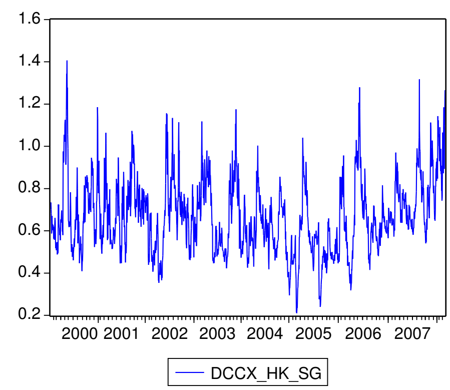
(c) Shanghai–Singapore



(d) Shenzhen–Hongkong



(e) Shenzhen–Singapore



(f) Hongkong–Singapore

Figure 13: Conditional correlation of DCCXI under Multivariate Normal Distribution

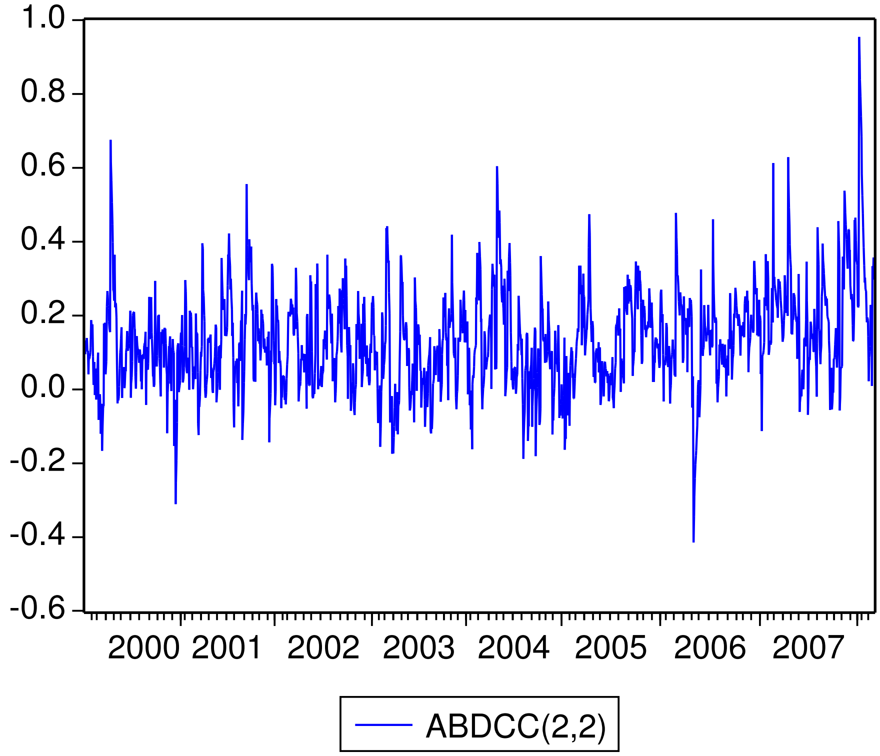


Figure 14: Conditional correlation between Block SH–SZ and Block HK–SG of AB-DCC(2,2) under Multivariate Normal Distribution

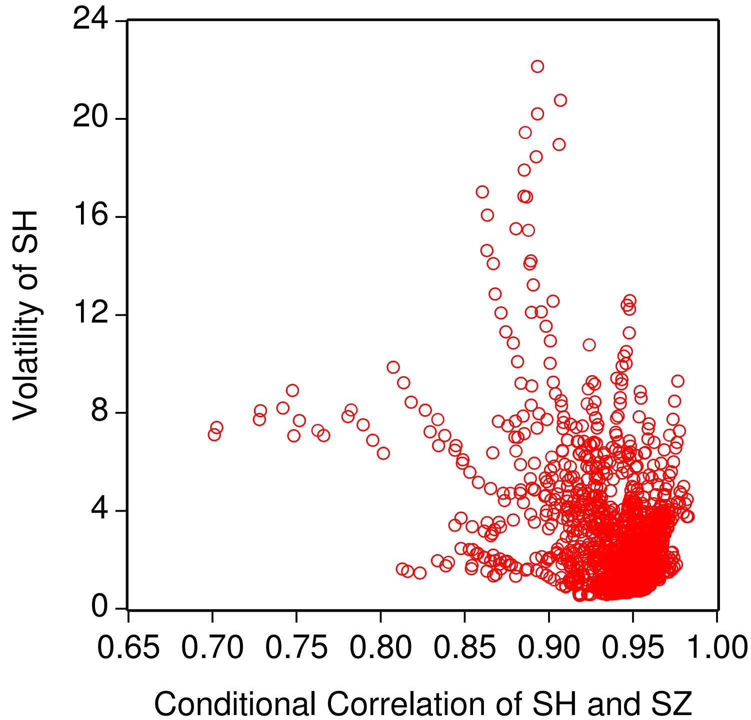


Figure 15: Scatter plot 1 between volatility and conditional correlations

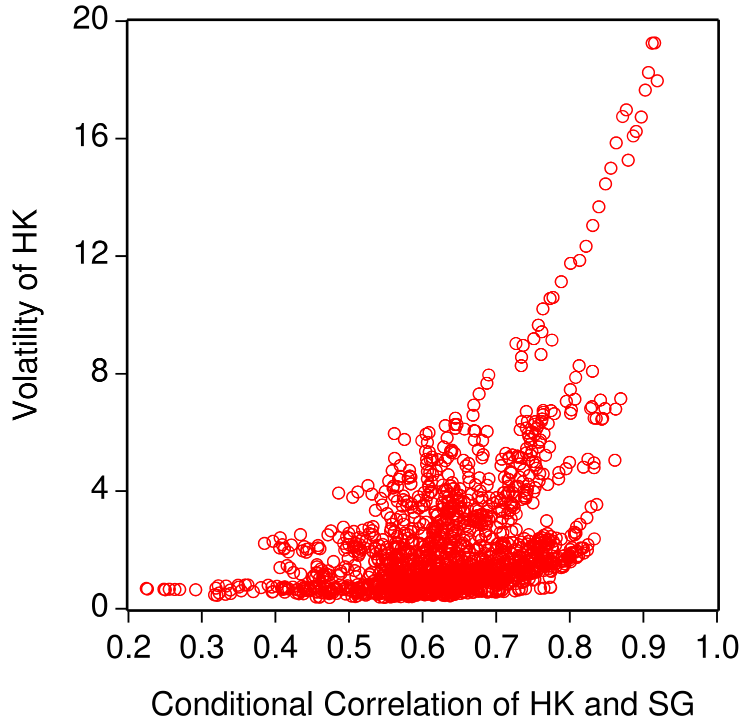


Figure 16: Scatter plot 2 between volatility and conditional correlations